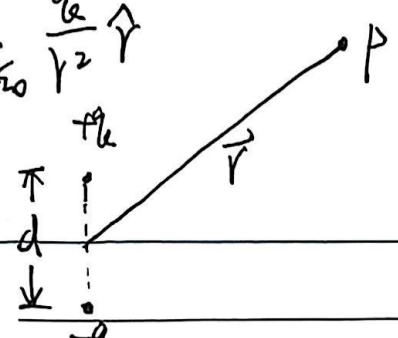


11

Saturday
农历四月初四
星期六

Lecture 1 Coulomb's Law and the Electric Field

1. quantized $q_e = ne$ $e = 1.602 \times 10^{-19} \text{C}$ 密立根油滴实验
2. charge is Lorentz invariant (洛伦兹不变量)
3. Coulomb's Law $\vec{F} = k \frac{q_1 q_2}{r^2} \hat{r} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^3} \vec{r}$ **vector**
 $k = 8.99 \times 10^9 \text{N}\cdot\text{m}^2/\text{C}^2$ $\epsilon_0 = 8.85 \times 10^{-12} \text{C}^2/\text{N}\cdot\text{m}^2$
4. 对导体: $T \uparrow, R \uparrow$ 对半导体: $T \uparrow, R \downarrow$
5. $\vec{E} = \frac{\vec{F}}{q_0}$ $q_0 \rightarrow 0$ 可视作质点
 $= \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$

 $\vec{P} = q \cdot \vec{d}$ (电偶极子大小)
 $\vec{E}_+ = \frac{1}{4\pi\epsilon_0} \frac{q}{r_+^2} \hat{r}_+ = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} (\hat{r} - \frac{\vec{d}}{2})$
 $\vec{E}_- = \frac{1}{4\pi\epsilon_0} \frac{-q}{r_-^2} \hat{r}_-$

12

Sunday
农历四月初五
母亲节星期日

$$|\vec{r}|^3 = [(\vec{r} - \frac{\vec{d}}{2}) \cdot (\vec{r} - \frac{\vec{d}}{2})]^{\frac{3}{2}} = (r^2 + \frac{d^2}{4} - \vec{r} \cdot \vec{d})^{\frac{3}{2}}$$

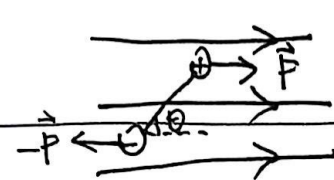
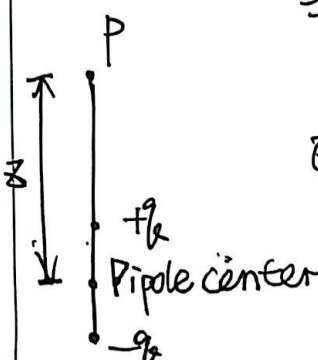
$$\vec{E} = \vec{E}_+ + \vec{E}_- = \frac{q}{4\pi\epsilon_0} \left[(r^2 + \frac{d^2}{4} - \vec{r} \cdot \vec{d})^{-\frac{3}{2}} (\vec{r} - \frac{\vec{d}}{2}) - (r^2 + \frac{d^2}{4} + \vec{r} \cdot \vec{d})^{-\frac{3}{2}} (\vec{r} + \frac{\vec{d}}{2}) \right]$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q}{r^3} \left[(1 + \frac{d^2}{4r^2} - \frac{\vec{r} \cdot \vec{d}}{r^2})^{-\frac{3}{2}} (\vec{r} - \frac{\vec{d}}{2}) - (1 + \frac{d^2}{4r^2} + \frac{\vec{r} \cdot \vec{d}}{r^2})^{-\frac{3}{2}} (\vec{r} + \frac{\vec{d}}{2}) \right]$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q}{r^3} \left[(1 + \frac{3\vec{r} \cdot \vec{d}}{2r^2}) (\vec{r} - \frac{\vec{d}}{2}) - (1 - \frac{3\vec{r} \cdot \vec{d}}{2r^2}) (\vec{r} + \frac{\vec{d}}{2}) \right]$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q}{r^3} (\frac{3\vec{r} \cdot \vec{d}}{r^2} \vec{r} - \vec{d}) = \frac{1}{4\pi\epsilon_0} \frac{1}{r^3} \left[\frac{3(\vec{P} \cdot \vec{r})}{r^2} \vec{r} - \vec{P} \right]$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{(z - \frac{d}{2})^2} - \frac{1}{4\pi\epsilon_0} \frac{q}{(z + \frac{d}{2})^2} = \frac{1}{2\pi\epsilon_0} \frac{P}{z^3}$$



$$\tau = -P d \sin \theta = -E P \sin \theta$$

2024 Lecture 2 Continuous charge and Gauss' Law

13

Monday
农历四月初六
星期一

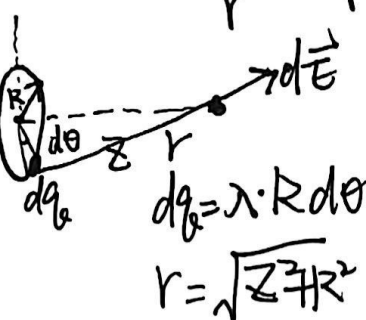
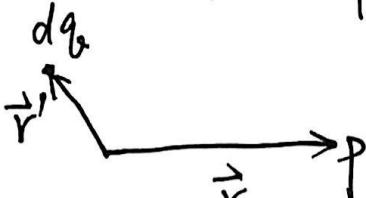
接受万物终将且必然走向死亡的事实，去无所羁，活到淋漓。

(圆环) check = 0 量纲

本周计划 TOP PRIORITIES

W20

☐ $\vec{E} = \int d\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{|\vec{r}-\vec{r}'|^3} (\vec{r}-\vec{r}')$



$d\vec{E}_z = \frac{1}{4\pi\epsilon_0} \frac{\lambda R d\theta}{(z^2 + R^2)^{3/2}} \cdot \frac{z}{\sqrt{z^2 + R^2}} \cdot \hat{z}$

$\vec{E} = \int d\vec{E}_z = \frac{1}{4\pi\epsilon_0} \frac{q}{(z^2 + R^2)^{3/2}} z \hat{z}$

$r = \sqrt{z^2 + R^2}$

圆盘 (Disk) 切成环叠加

① 取极限 (i) $z \rightarrow 0$ $E \rightarrow 0$.

(ii) $z \rightarrow \infty$ $\lim_{z \rightarrow \infty} \frac{z}{(z^2 + R^2)^{3/2}} = \lim_{z \rightarrow \infty} \frac{1}{(1 + \frac{R^2}{z^2})^{3/2}} \cdot \frac{1}{z^2} = \frac{1}{z^2}$

无穷小

$ds = 2\pi r dr$
 $d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{\sigma 2\pi r dr}{(z^2 + r^2)^{3/2}} z \hat{z}$
 $= \frac{\sigma}{2\epsilon_0} z \hat{z} \left[\frac{1}{z} - \frac{1}{(z^2 + r^2)^{1/2}} \right]$

14

Tuesday
农历四月初七
星期二

$\lim_{z \rightarrow \infty} \left[1 - \frac{z}{(z^2 + r^2)^{1/2}} \right] = \lim_{z \rightarrow \infty} \left[1 - \frac{1}{(1 + \frac{r^2}{z^2})^{1/2}} \right] = 1 - (1 - \frac{r^2}{2z^2}) = \frac{r^2}{2z^2}$

☐ $R \rightarrow \infty$ 无限大平板 $E = \frac{\sigma}{2\epsilon_0}$

☐ 电通量 Electric Flux $\Phi = \int \vec{E} \cdot d\vec{S}$ 面的法线方向的面

☐ 高斯定理 (对于闭合曲面) $\oint \vec{E} \cdot d\vec{S} = \frac{q_{enc}}{\epsilon_0}$ 面的内面.

对于球 $\oint \vec{E} \cdot d\vec{S} = \oint \epsilon_0 \frac{q}{r^2} \hat{r} \cdot d\vec{S} = \oint \epsilon_0 \frac{q}{r^2} ds$
 $\oint ds = 4\pi r^2$

库仑定律只在静电场中适用. 高斯定理恒成立

$\rho = \frac{q}{V}$ $\nabla \cdot (\nabla V) = \nabla^2 V = \Delta V = -\frac{\rho}{\epsilon_0}$ $\frac{\partial^2}{\partial x^2} V + \frac{\partial^2}{\partial y^2} V + \frac{\partial^2}{\partial z^2} V = -\frac{\rho}{\epsilon_0}$

$\oint \vec{A} \cdot d\vec{S} = \int (\nabla \cdot \vec{A}) dV \rightarrow$ 体积

$\oint \vec{E} \cdot d\vec{S} = q$

高斯定理微分形式: $\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$

① 对于球对称体系 $E(r) \cdot S = \frac{q}{\epsilon_0}$

对于柱状体系 $\oint \vec{E} \cdot d\vec{S} = \int E(\rho) ds = E(\rho) 2\pi \rho h = \frac{\lambda h}{\epsilon_0}$

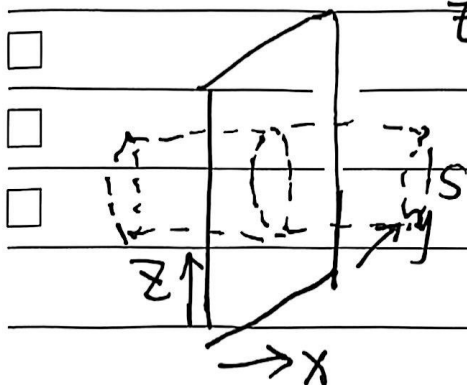
($\vec{E} = E(\rho) \hat{\rho}$)

$\therefore E(\rho) = \frac{\lambda}{2\pi\epsilon_0 \rho}$

上下底面 $\Phi \rightarrow 0$.

15 Wednesday
农历四月初八
星期三

无限大平板



$$\vec{E}(\vec{r}) = \hat{x} E(x) \quad \oint \vec{E} \cdot d\vec{S} = 2E(x) \cdot s = \frac{\sigma s}{\epsilon_0}$$

$$\therefore E(x) = \frac{\sigma}{2\epsilon_0}$$

注: 运用高斯定理时, 所选开尔文面必须具有对称性极强 (即上述球柱面) 时, 其他开尔文面需用库仑定律积分。

Lecture 3 The Electrostatic Potential

保守性 做功与路径无关 (在闭合路径上做功为0)
 $\oint \vec{F} \cdot d\vec{r} = 0$ 判断可用交叉求导 eg. $\vec{F} = 2y\hat{i} + x^2\hat{j}$ 此时积分下
 (复变函数在闭合回路上的积分等于复变函数在圆内为0)

路内函数之和除以2π)

$$\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2} = 0$$

16

Thursday
农历四月初九
星期四

斯托克斯定理

$$\oint \vec{A} \cdot d\vec{r} = \int (\nabla \times \vec{A}) \cdot d\vec{S}$$

$\nabla \cdot \vec{A}$ 散度	$\hat{x}$	$\hat{y}$	$\hat{z}$
$\nabla \times \vec{A}$	$\frac{\partial}{\partial x}$	$\frac{\partial}{\partial y}$	$\frac{\partial}{\partial z}$
	A_x	A_y	A_z

在闭合路径C 旋度以C为边界的面积

对保守性 $\nabla \times \vec{F} = 0$ (在三维空间)

四维点成立)

Q $\vec{r} = r\hat{r}$ $d\vec{r} = dr\hat{r}$

$$W = \int_A^B \vec{F} \cdot d\vec{r} = \int_A^B \frac{1}{4\pi\epsilon_0} \frac{qq}{r^2} \hat{r} \cdot d\vec{r}$$

$$\hat{r} \cdot d\vec{r} = |\vec{r} + d\vec{r}| - |\vec{r}| = dr$$

$$W = \int_A^B \frac{1}{4\pi\epsilon_0} \frac{qq}{r^2} dr = \frac{qq}{4\pi\epsilon_0} \left(\frac{1}{r_A} - \frac{1}{r_B} \right)$$

首先定义电势零点, (无限远处可作电势零点)

点, 但只对有限电荷分布适用)

取 ∞ 处为电势零点 $\Delta U = -W = -\int_i^f \vec{F} \cdot d\vec{s} = q_0 \int_i^f \vec{E} \cdot d\vec{s}$

$$V(\vec{r}) = \int_{\infty}^{\vec{r}} \vec{E} \cdot d\vec{s}$$

$$U_f - U_i = q_0 \int_i^f \vec{E} \cdot d\vec{s}$$

电势 (电场本身的外生质)

17

Friday
农历四月初十
星期五

就是因为想不通、不了解、没有够多的钱、也没有够多的朋友，那才叫成长，你才会面对和解决它。

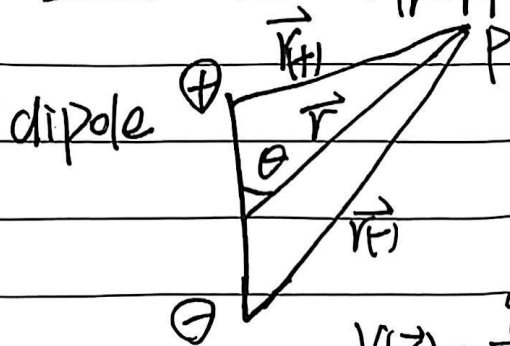
□ 对于电场 $V_f - V_i = -E \cdot \Delta x$

□ $V = C$. Equipotential surface 等势面 (与电场线处处垂直)

□ 对于球壳: $V = \int_r^\infty \frac{q}{4\pi\epsilon_0 r^2} dr = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$

$V = \sum \frac{1}{4\pi\epsilon_0} \frac{q_i}{r_i} = \frac{1}{4\pi\epsilon_0} \sum \frac{q_i}{|\vec{r} - \vec{r}_i|}$ $\vec{r}$ 为所求空间位置

$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{|\vec{r} - \vec{r}'|}$ $dq = \rho(\vec{r}') dV$ $\vec{r}_i$ 为点电荷所在位置



$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r_+} - \frac{q}{r_-} \right)$$

$$r_+ = \sqrt{r^2 + \frac{d^2}{4} - rd \cos \theta}$$

$$r_- = \sqrt{r^2 + \frac{d^2}{4} + rd \cos \theta}$$

$$V(\vec{r}) = \frac{q}{4\pi\epsilon_0} \left[\left(r^2 + \frac{d^2}{4} - rd \cos \theta \right)^{-\frac{1}{2}} - \left(r^2 + \frac{d^2}{4} + rd \cos \theta \right)^{-\frac{1}{2}} \right]$$

18

Saturday
农历四月十一
星期六

$\therefore r \gg d \quad \therefore d \ll r \quad V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{q}{r} \left(1 + \frac{d^2}{4r^2} - \frac{d \cos \theta}{r} \right)^{-\frac{1}{2}}$

$\left(1 + \frac{d^2}{4r^2} - \frac{d \cos \theta}{r} \right)^{-\frac{1}{2}}$

$$= \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} d \cos \theta = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \vec{r}}{r^3}$$

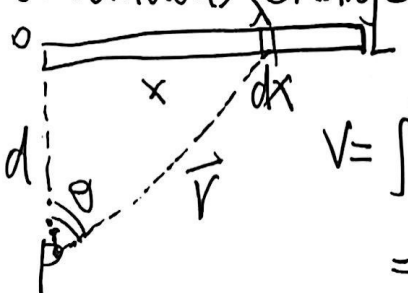
一堆正电荷构成的体系所具有的能量: $U_{total} = \frac{1}{4\pi\epsilon_0} \sum_{i < j} \frac{q_i q_j}{r_{ij}}$

$$= \frac{1}{2} \cdot \frac{1}{4\pi\epsilon_0} \sum_{i \neq j} \frac{q_i q_j}{r_{ij}}$$

$$= \frac{1}{2} \frac{1}{4\pi\epsilon_0} \iint \frac{\rho(\vec{r}) \rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3\vec{r} d^3\vec{r}'$$

对于连续电荷分布 $\vec{r}$ 和 $\vec{r}'$

Continuous Charge Distribution



$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r}$$

$$dq = \lambda dx \quad r = \sqrt{x^2 + d^2}$$

$$V = \int_0^L \frac{1}{4\pi\epsilon_0} \frac{\lambda dx}{\sqrt{x^2 + d^2}}$$

$$\text{令 } x = d \tan \theta$$

$$= \frac{\lambda}{4\pi\epsilon_0} \int \frac{d\theta}{\cos \theta} = \frac{\lambda}{4\pi\epsilon_0} \int \frac{d \sin \theta}{1 - \sin^2 \theta}$$

$$\int \frac{d \sin \theta}{1 - \sin^2 \theta} = \int \frac{1}{2} \left(\frac{1}{1 - \sin \theta} + \frac{1}{1 + \sin \theta} \right) d \sin \theta = \frac{1}{2} \ln \left| \frac{1 + \sin \theta}{1 - \sin \theta} \right| \Big|_0^{\frac{L}{\sqrt{L^2 + d^2}}}$$

$$= \ln \frac{L + \sqrt{L^2 + d^2}}{d}$$

$$\therefore V = \frac{\lambda}{4\pi\epsilon_0} \ln \frac{L + \sqrt{L^2 + d^2}}{d}$$

19

Sunday
农历四月十二
星期日

对于无限长杆不可将无穷远点视为电势零点

λ
半无限长. 除以2.

Disk

$E(r) = \frac{\lambda}{2\epsilon_0 r}$ $V = \int_r^{\infty} \frac{\lambda}{2\epsilon_0 r} dr$

$dq = \sigma 2\pi R' dR'$
 $r = \sqrt{z^2 + R'^2}$

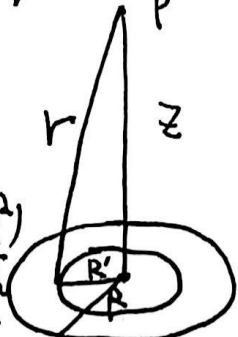
$V = \int_0^R \frac{1}{4\pi\epsilon_0} \frac{\sigma 2\pi R' dR'}{\sqrt{z^2 + R'^2}} = \frac{\sigma}{2\epsilon_0} \int_0^R \frac{R' dR'^2 + z^2}{\sqrt{R'^2 + z^2}}$

$= \frac{\sigma}{2\epsilon_0} \sqrt{R'^2 + z^2} \Big|_0^R = \frac{\sigma}{2\epsilon_0} (\sqrt{R^2 + z^2} - z)$

$-\frac{dV}{dz} = \frac{\sigma}{2\epsilon_0} (1 - \frac{z}{\sqrt{R^2 + z^2}}) = E_z$

$\oint \vec{E} \cdot d\vec{s} = \oint [V_0 - (V_0 + \delta V)] \quad \vec{E} d\vec{s} = -\delta V$

$\vec{E} = -\nabla V$ 是保守场的充要条件: $\vec{F} = f(r)\hat{r}$.



20

Monday
农历四月十三
星期一

梦想的确使人勇敢和伟大, 但是妄想和想象力是两回事.

Lecture 4 The Triangle of Electrostatics

本周计划 TOP PRIORITIES

W21

- ☐ Calculate the electric field: 库仑定律
- ☐ $\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \hat{r}$
- ☐ ②高斯 ③电势 $\vec{E} = -\nabla V$
- ☐ $\nabla \cdot \vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$
- ☐ 静电场性质: $\nabla \times \vec{E} = 0$
- ☐ $\nabla \times (\nabla V) = 0$
- ☐ $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$

左面电通量 $\vec{E}(x-\frac{\Delta x}{2}, y, z) \cdot \Delta \vec{s}$
 $= -E_x(x-\frac{\Delta x}{2}, y, z) \Delta y \Delta z$

右: $E_x(x+\frac{\Delta x}{2}, y, z) \Delta y \Delta z$

$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho}{\epsilon_0}$ 电荷密度

$\vec{E} = -\nabla V$

$V = -\int \vec{E} \cdot d\vec{r}$

$\oint \vec{E} \cdot d\vec{s} = \frac{q_{enc}}{\epsilon_0} = \frac{\rho(x,y,z) \Delta x \Delta y \Delta z}{\epsilon_0}$

中位置 (x, y, z)

$(\vec{E} \cdot \Delta \vec{s})_x = \frac{\partial E_x}{\partial x} \Delta x \Delta y \Delta z$

$\therefore \frac{\rho(x,y,z) \Delta x \Delta y \Delta z}{\epsilon_0} = (\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}) \Delta x \Delta y \Delta z \quad \nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$

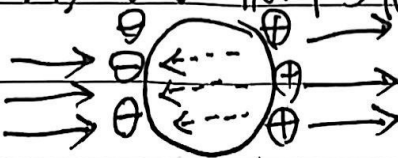
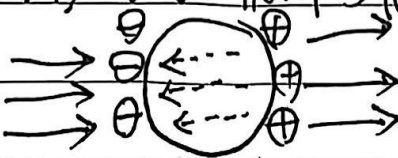
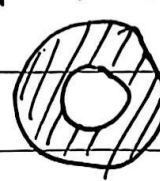


21

Tuesday
农历四月十四
星期二

$$\oint_S \vec{v} \cdot d\vec{A} = \int_V (\nabla \cdot \vec{v}) dV$$

Lecture 5 The Electrical Properties of Conductors

- 波粒二象性 $m \uparrow \lambda \downarrow$ 德布罗意公式 $\lambda = \frac{h}{p}$
- 静电平衡 $\Rightarrow$  $E_{\text{外}} = E_{\text{内}}$
- 内部为等势体 $\Rightarrow$  内部电场处处为0.
- 导体内部任一高斯面的电通量 $\Rightarrow$ 导体内部不存在净电荷.
- 导体内部存在空腔  空腔内部若无其他电荷, 则为表面不带电. 若有则为表面带等量反号电荷.

静电平衡并不要求 'perfect' 导体

shell theorem 对于球状导体可等效为中心点电荷.

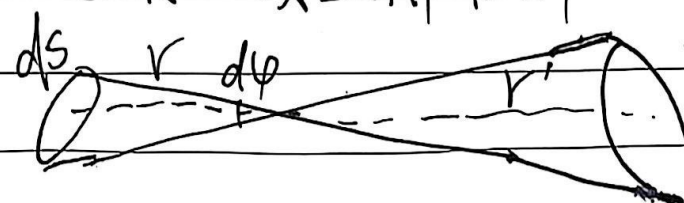
若观测点在球壳内部, 若球壳上电荷均匀分布, 则该点电场为0.

22

Wednesday
农历四月十五
星期三

点电荷为0.

证明: 设立体角为 $d\Omega$ (立体角的定义: 以锥体的顶

-  ds r $d\Omega$
- ds' 点为球心作球面, 该锥体在球表面截取的面积与球半径平方成正比

$$dq = \sigma \cdot ds = \sigma \cdot r^2 d\Omega / \cos \theta$$

$$dq' = \sigma \cdot ds' = \frac{\sigma r'^2 d\Omega}{\cos \theta'}$$

$\theta = \theta'$
 $dq \propto r^2$ $dq' \propto r'^2$

求解正方体电荷分布, 带电量为 q : 正方体内部等势, 无穷远处电势为0. 用 $\nabla^2 V = -\rho/\epsilon_0$ 求 V . 用 $\vec{E} = -\nabla V$ 求 $\vec{E}$.

表面电场与表面电荷垂直. 表面取一高斯面. 电通量仅由上表面贡献.

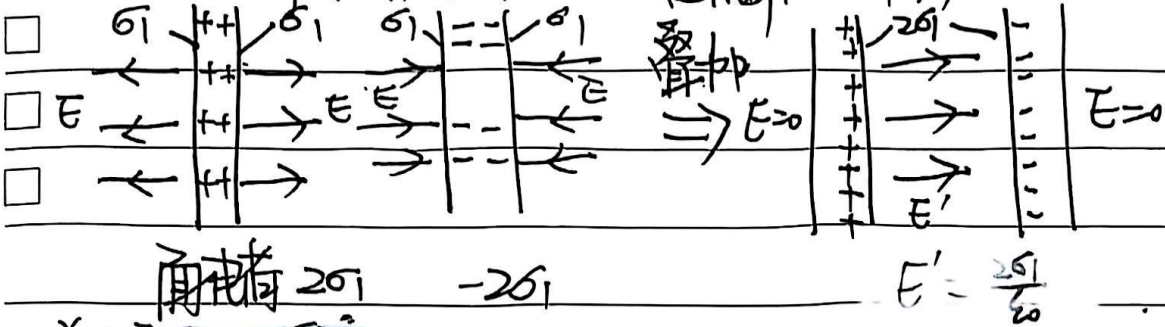
 $\vec{E} \cdot d\vec{S} = \frac{\sigma \cdot dS}{\epsilon_0}$ $\vec{E} = \frac{\sigma}{\epsilon_0} \hat{n}$

23

Thursday
农历四月十六
星期四

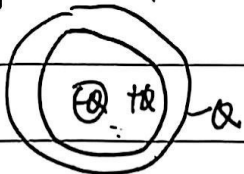
曲率半径越大,面电荷密度 σ 越小

Parallel Plates (金属板, 无穷大)



单独: $E = \frac{\sigma}{\epsilon_0}$

球壳内部有电荷: 不论电荷位于何处, 外表面电荷均匀分布。



静电屏蔽不仅可以屏蔽外部对内部的影响, 也可以屏蔽内部对外部的影响。

induce charge 感应电荷。

求解电荷分布时: 对于 $\nabla^2 V = -\rho/\epsilon_0$ 已知电荷分布与边界条

24

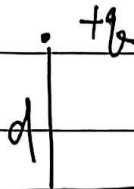
Friday
农历四月十七
星期五

每个人依然会在坦然和容易之间做选择, 但一直选择坦然, 也是不容易的。

件即同向一确定

如某面约束值已知
或面上所偏导(场强)已知

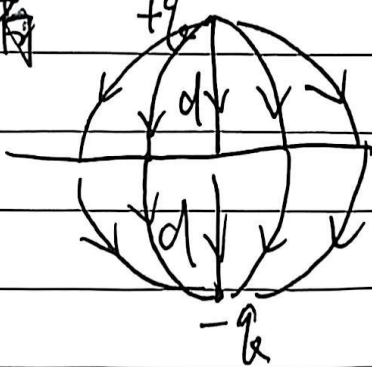
e.g.
□
□
□



求电场、电势分布。

→ 无限大平板(接地)

镜像电荷



(若接地板
下无电荷
分布)

$$V_+ = \frac{1}{4\pi\epsilon_0} \frac{q}{\sqrt{x^2+y^2+(z-d)^2}}$$

$$V = V_+ + V_- \quad z \geq 0$$

$$V = 0 \quad z < 0$$

$$q d \frac{1}{4\pi\epsilon_0} \left(\frac{1}{\sqrt{x^2+y^2+(z-d)^2}} - \frac{1}{\sqrt{x^2+y^2+(z+d)^2}} \right)$$

$$\sigma = -\epsilon_0 \frac{\partial V}{\partial z} \Big|_{z=0} = -\frac{1}{2\pi} \frac{q d}{(x^2+y^2+d^2)^{3/2}} = -\frac{q d}{2\pi (r^2+d^2)^{3/2}}$$

板上带电量

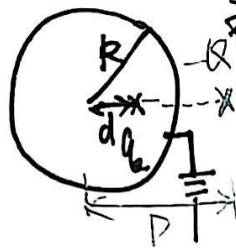
$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sigma dx dy = -q$$

$$= \int \frac{-q d}{2\pi (r^2+d^2)^{3/2}} 2\pi r dr = \int_0^{\infty} \frac{-q d}{2(r^2+d^2)^{3/2}} d(r^2+d^2) = -q$$

25

Saturday
农历四月十八
星期六

点电荷从无穷远到平板靠近需克服电场力做功

对于球壳
R为半径求内表面电荷分布.
用镜像电荷.

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{\sqrt{(x-d)^2 + y^2 + z^2}} - \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{(x-d)^2 + y^2 + z^2}}$$

$$V=0 \text{ 与 } x^2 + y^2 + z^2 = R^2 \text{ 等价}$$

$$\therefore Q^2 [(x-d)^2 + y^2 + z^2] = q^2 [(x-d)^2 + y^2 + z^2]$$

$$(Q^2 - q^2)(x^2 + y^2 + z^2) - 2Q^2 dx + q^2 dx = q^2 R^2 - Q^2 d^2$$

阿波罗尼斯圆: $\frac{Q^2}{q^2} = \frac{d^2}{R^2}$ 且 $\frac{Q^2 d^2 - Q^2 R^2}{Q^2 - q^2} = R^2$

Lecture 6 Resistance and Capacitance

26

Sunday
农历四月十九
星期日 $i = dq/dt$ 标量稳态 $i_1 = i_2$
 i 不随 t 变化

$$i_1 = i_2$$

$$q = \int dq = \int_{t_1}^{t_2} i dt$$

 $i = \int \vec{j} \cdot d\vec{A}$ $\vec{j}$ 为电流密度 载流子运动方向即为 $\vec{j}$ 方向.能均分定理 每一个自由度对平均能量有 $\frac{1}{2} k_B T$ 贡献.热运动平均速度与温度关系 电子自由度为 3. $\frac{3}{2} k_B T = \frac{1}{2} m_e \bar{v}^2$

$$\bar{v} = \sqrt{\frac{3k_B T}{m_e}} \approx 3 \times 10^4 \text{ m/s.}$$

$$\text{漂移速度 } v_d \text{ 很小 (较之 } \bar{v} \text{ 小)} = nevd \text{ or } i = \frac{q}{dt} = nAev_d$$

drift velocity

 n 为载流子浓度 (单位体积内移动的电子数)e.g. 求 v_d 有一 $1\text{mm} \times 1\text{mm}$ 的长方体铜导线. 通 1A 电流 $J = \frac{i}{A}$. 铜晶胞 面心+顶角 1立方体有4个Cu原子. $3d^{10} 4s^1$ 

每个铜原子提供1个载流子

27

Monday
农历四月二十
星期一

老的可怕不在于老这件事本身，而在于因为老丧失了生命力和改变自己的意志。

R 标准定义 $R = \frac{V}{I}$ 微分定义 $R = \frac{dV}{dI}$

本周计划 TOP PRIORITIES

W22

☐ $R = \frac{\rho L}{S}$ 电阻率 $\sigma = \rho^{-1}$

☐ 欧姆定律: R 与 V, I 无关

☐ 碰撞的平均效应: 电子

☐ 碰撞完静止 $\vec{a} = -\frac{e\vec{E}}{m}$

☐ $\vec{v}_d = \vec{a}\tau = -\frac{e\vec{E}}{m}\tau$ mean free time

☐ $\vec{J} = n(-e)\vec{v}_d$ $\vec{E} = \rho\vec{J}$

☐ $\rho = \frac{m}{ne^2\tau}$ ρ 为电阻率

☐ L $I = \frac{VA}{\rho L} = \frac{V}{L} \frac{A}{\rho} = \frac{EA}{\rho} = JA$
 $\therefore E = \rho J$ $J = \sigma E$

Equation of Continuity

$\oint \vec{J} \cdot d\vec{A} = -\frac{dq}{dt} = -\int \frac{\partial \rho}{\partial t} dV$

$\oint \vec{V} \cdot d\vec{A} = \int (\nabla \cdot \vec{V}) dV$ (电荷守恒)

$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$ 电流的连续性方程

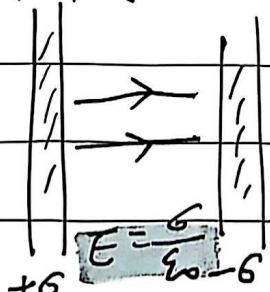
电容首先判断是否为极板

$\sigma = \frac{q}{A}$

$E = \frac{\sigma}{\epsilon_0 A}$

$V = Ed$

$C = \frac{q}{V} = \frac{\epsilon_0 A}{d}$



28

Tuesday
农历四月廿一
星期二

C 串联 = $\frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}}$ C 并联 = $C_1 + C_2 + C_3 + \dots + C_n$

☐ $C = \frac{\epsilon_r \epsilon_0 S}{4\pi k d} = \frac{\epsilon_r \epsilon_0 S}{d}$ ϵ_r 为相对介电常数, 无单位

☐ 电容器刚接入电路时可视为短路

☐ 定义 $C = \frac{Q}{V}$ 中 Q 为带电量, 指一个极板所带电荷量的绝对值。

二维向量旋度: $\nabla \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & 0 \end{vmatrix} = \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \hat{k}$

求同轴电缆电容

假设长为 L ($L \gg b$) $\lambda = \frac{q}{L}$

验证: 平行板电容器

高斯面为圆柱体, 高为 h.

$a \rightarrow \infty$ $b - a = d$

$2\pi r h E = \frac{\lambda h}{\epsilon_0}$

$E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r}$

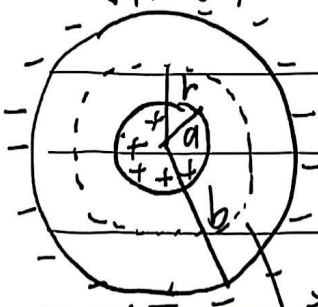
$2\pi\epsilon_0 L \lim_{b \rightarrow \infty} \frac{1}{\ln \frac{b}{a}}$

$V = \int_a^b \vec{E} \cdot d\vec{r} = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{b}{a}$

$= 2\pi\epsilon_0 L \lim_{a \rightarrow \infty} \frac{1}{\ln \left(1 + \frac{d}{a} \right)}$

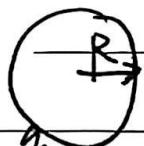
$C = \frac{q}{V} = 2\pi\epsilon_0 L / \ln \frac{b}{a}$

$= 2\pi\epsilon_0 L \frac{a}{d}$



高斯面

29

Wednesday
农历四月廿二
星期三孤立球形导体 $C = 4\pi\epsilon_0 R$ 将无穷远处视为另一极板.☐

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \uparrow \quad V = \frac{1}{4\pi\epsilon_0} \frac{Q}{R} \quad C = 4\pi\epsilon_0 R$$

☐Lecture 7 DC circuits☐emf device 电势差 描述 $\mathcal{E}$ $\uparrow$
接法均可视作用导线连接

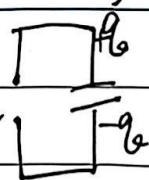
$$\text{电势差做功} \quad dw = \mathcal{E} dq \quad P_{\mathcal{E}} = \frac{dw}{dt} = \mathcal{E} \frac{dq}{dt} = i\mathcal{E}$$

$$i^2 R = P_R = P_{\mathcal{E}} = i\mathcal{E} \quad \mathcal{E} - iR = 0$$

基尔霍夫环路定理 Kirchhoff's loop rule: 沿回路环绕一周回到出发点, 电压降为0.

基尔霍夫电流定理 $\rightarrow$ 流入为+, 流出为-.

30

将 dq 从正极板拉到正板板

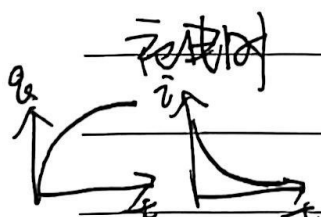
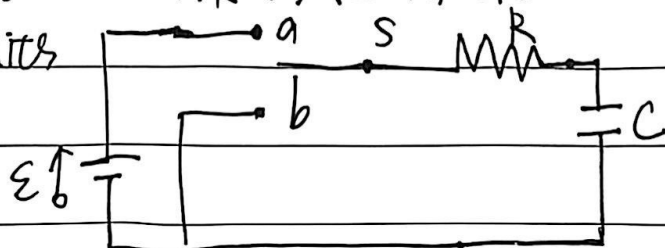
$$dw = \mathcal{E} dq \quad \mathcal{E} = \frac{q}{C}$$

$$d\mathcal{E}C = dw = \frac{q}{C} dq$$

$$\mathcal{E}C = \int \frac{q}{C} dq = \frac{q^2}{2C} = \frac{1}{2} CV^2$$

☐☐ $V_{ab} = 0$ 可视为作用导线连接 a, b.☐

RC Circuits



$$\mathcal{E} - iR = \frac{q}{C} = 0$$

$$\mathcal{E} = R \frac{dq}{dt} + \frac{q}{C}$$

$$i_0 = \frac{dq}{dt} + \frac{q}{C}$$

$$q = \mathcal{E}C(1 - e^{-t/\tau})$$

$$\frac{\mathcal{E}}{R} = \frac{dq}{dt} + \frac{q}{RC}$$

$$(i_0 - \frac{q}{RC}) + dq = dt \quad \text{两边积分}$$

$$i = \frac{\mathcal{E}}{R} e^{-t/\tau}$$

放电时.

$$\frac{q}{C} - iR = 0$$

$$i = -\frac{dq}{dt}$$

$$\frac{q}{C} + R \frac{dq}{dt} = 0$$

$$\frac{q}{C} + \frac{dq}{dt} = 0$$

$$q = q_0 e^{-t/\tau}$$

$$i = \frac{q_0}{RC} e^{-t/\tau} = \frac{\mathcal{E}}{R} e^{-t/\tau}$$

31

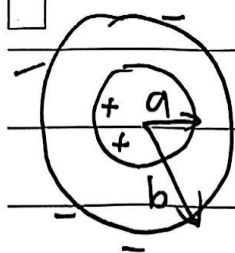
Friday
农历四月廿四
星期五

明天我们将不理解今天的执着, 就像今天的我们不再理解儿童时代为何执着地要那件玩具一样。

□ 由 $C = \frac{\epsilon_0 A}{d}$ $E_c = \frac{1}{2} \epsilon_0 V^2 = \frac{1}{2} \frac{\epsilon_0 A}{d} V^2 = \frac{1}{2} \epsilon_0 A d \left(\frac{V}{d}\right)^2 = \frac{1}{2} \epsilon_0 A d E^2$

□ 电场能量密度 $W_E = \frac{1}{2} \epsilon_0 E^2$ 对任何场均成立

□ a, b 分别为球外半径, 内半径, 求 C.



法一: 对于半径为 r 的高斯面

$$Q = \epsilon_0 E A = \epsilon_0 E 4\pi r^2 \quad E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

$$V = \int_a^b \vec{E} \cdot d\vec{r} = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b}\right) = \frac{Q}{4\pi\epsilon_0} \frac{b-a}{ab}$$

$$C = \frac{Q}{V} = 4\pi\epsilon_0 \frac{ab}{b-a}$$

法二: 半径为 a 的球与无穷远处构成电容 $C_1 = 4\pi\epsilon_0 a$

b

$$C_2 = 4\pi\epsilon_0 b$$

C 与 C_2 串联得 C, $\therefore C^{-1} = C_1^{-1} + C_2^{-1} \therefore C = 4\pi\epsilon_0 \frac{ab}{b-a}$

本月总结

法三: $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}$ $W_E = \frac{1}{2} \epsilon_0 E^2 = \frac{1}{32\pi^2 \epsilon_0} \frac{Q^2}{r^4}$

$$E_c = \int W_E dV = \int_a^b \frac{1}{32\pi^2 \epsilon_0} \frac{Q^2}{r^4} 4\pi r^2 dr = \frac{1}{8\pi\epsilon_0} Q^2 \left(\frac{1}{a} - \frac{1}{b}\right) = \frac{Q^2}{2C}$$

绝对电介常数

有介电: $\epsilon = k \epsilon_0$ $E = \frac{1}{4\pi\epsilon} \frac{Q}{r^2} = \frac{1}{4\pi k \epsilon_0} \frac{Q}{r^2}$

$$E = \frac{Q}{k \epsilon_0 A} = \frac{E_0}{k} = \frac{Q/k}{\epsilon_0 A}$$

$$\oint k \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$$\oint \vec{D} \cdot d\vec{A} = Q$$

电位移矢量 $\vec{D} = k \epsilon_0 \vec{E}$

k 为相对介电常数

电介质本身为绝缘体只有正负电荷中心偏移, 而无电荷的聚集 (束缚电荷)

Lecture 8 The Magnetic Field

$$\vec{F}_B = q \vec{v} \times \vec{B}$$

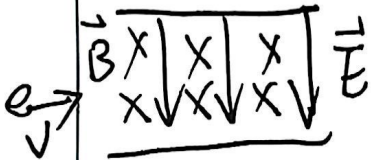
洛伦兹力不做功 $P = \vec{F} \cdot \vec{v}$ $P_B = 0$

1

Saturday
农历四月廿五
儿童节星期六

$B_{\text{地}} = 10^4 \text{ gauss}$ (地磁场的 1 gauss)

地理北极为地磁南极



$$\vec{B} = B\hat{z} \quad \vec{E} = E\hat{y} \quad \vec{v} = v_x\hat{x} + v_y\hat{y}$$

$$F = -e\vec{E} + (-e)\vec{v} \times \vec{B}$$

$$= -eE\hat{y} - e(v_x\hat{x} + v_y\hat{y}) \times B\hat{z}$$

$$= e(v_x B - E)\hat{y} - e v_y B \hat{x}$$

$$= m \left(\frac{dv_x}{dt} \hat{x} + \frac{dv_y}{dt} \hat{y} \right)$$

$$eB \frac{dv_x}{dt} = m \frac{d^2 v_y}{dt^2}$$

$$\frac{dv_x}{dt} \text{ 即可用 } \frac{m}{eB} \frac{d^2 v_y}{dt^2} \text{ 代入}$$

$$\frac{d^2 v_y}{dt^2} + \left(\frac{eB}{m} \right)^2 v_y = 0$$

$$\begin{cases} e(v_x B - E) = m \frac{d}{dt} v_y \\ -e v_y B = m \frac{d}{dt} v_x \end{cases}$$

解此二元一次微分方程组

用一个代换另一个由于A的
左边有常数对其求导的
微分可使形式简便

2

Sunday
农历四月廿六
星期日

$$\frac{eB}{m} = \omega$$

$$v_y = A \cos(\omega t + \phi)$$

$$v_x = A \sin(\omega t + \phi) + \frac{E}{B} \quad (\text{漂移速度})$$



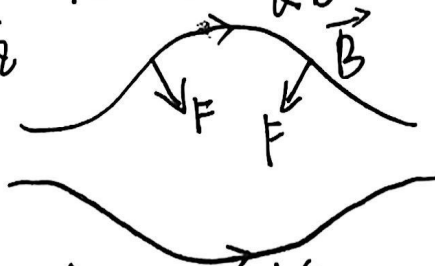
$$q v B = \frac{m v^2}{R}$$

磁瓶

$$R = \frac{m v}{q B}$$

$$T = \frac{2\pi R}{v} = \frac{2\pi}{\omega}$$

应用: 托卡马克



回旋加速器: 速度越大(接近光速) 质量越大(相对论)

$$T = \frac{2\pi}{\omega} = \frac{2\pi m}{q B}$$

B 稍微变化

3

Monday
农历四月廿七
星期一

每一个伤口都会愈合。只要反省总结，受过伤的地方就会金刚不坏。

本周计划 TOP PRIORITIES

W23

☐ 霍尔效应

☐

☐ $E_H = V_H$

☐ $V_H = \frac{IB}{nq_d h}$

☐ $E = vB$ 平衡 $j = nqv$

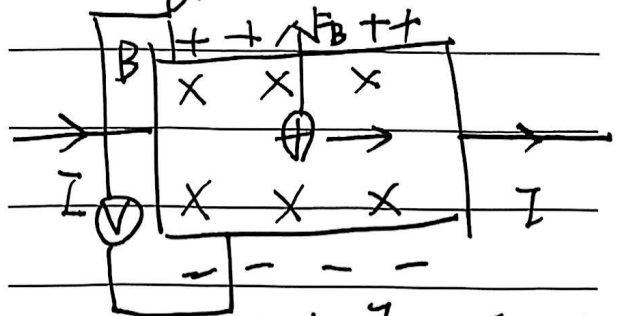
☐ $\frac{V_H}{d} = \frac{j}{nq} \cdot B = \frac{I}{dh}$

☐ $\frac{V_H}{d} = \frac{I}{nq_d h} \cdot B$

☐ $V_H = \frac{IB}{nq_h}$ 形式与 $V = IR$ 一致

霍尔系数

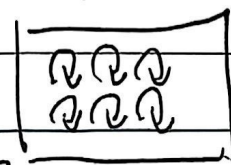
$$R_H = \frac{B}{nq} \text{ 霍尔电阻}$$



上极板电势高于下极板

霍尔效应

$$R = \frac{m}{qB} \quad B \uparrow \quad R \downarrow$$



中间绝缘导线

4

Tuesday
农历四月廿八
星期二

磁场对电流的作用

$$d\vec{F} = i(d\vec{l}) \times \vec{B}$$

☐ $\vec{F}_B = e\vec{v} \times \vec{B} \quad |q| = i(L/Vd)$

☐ $\vec{F} = \frac{iL}{Vd} \frac{e}{|e|} \vec{v} \times \vec{B} = i\vec{L} \times \vec{B}$

☐ 安培力的做功 (洛伦兹力不做功, 电源为了维持电流做功)

以 CD 为轴

$$\vec{\tau} = \vec{r} \times \vec{F}_{AB} = \vec{DA} \times (I\vec{AB} \times \vec{B})$$

$$= I\vec{DA} \times (\vec{AB} \times \vec{B})$$

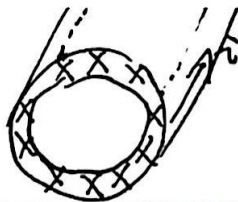
$$= I(\vec{DA} \times \vec{AB}) \times \vec{B}$$

$\vec{DA} \times \vec{AB}$: 矩形面积(大小) 叉乘无结合律 仅该情形下恰好成立

以电流右手螺旋规定的正方向(方向)

视作 $\vec{S}$ $I\vec{S} = \vec{\mu}$ 磁偶极子 $\vec{\tau} = \vec{\mu} \times \vec{B}$

2024



可等效为



June

5

Wednesday

农历四月廿九

芒种 星期三

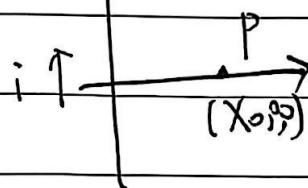
Lecture 9 Magnetic Field of a Current

磁场和电场的相对论效应 (磁场为无源场)

□ 电生磁: 运动的电荷产生磁场

□ Biot-Savart Law $dB = \frac{\mu_0}{4\pi} \frac{id\vec{s} \times \vec{r}}{r^3}$ $\mu_0 = 4\pi \times 10^{-7} \text{ Tm/A}$ □ 例: 无限长直导线 $dB = \frac{\mu_0}{4\pi} \frac{id\vec{y} \hat{y} \times \vec{r}_0 \hat{x}}{r^3}$

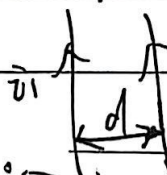
(0, y)



$$= \frac{\mu_0}{4\pi} \frac{id\vec{y}(-\hat{z}) \cdot x_0}{r^3}$$

$$\vec{B} = \int_{-\infty}^{+\infty} \frac{\mu_0 i}{4\pi} \frac{x_0 dy(-\hat{z})}{(x_0^2 + y^2)^{3/2}} \quad \text{令 } y = x_0 \tan \theta$$

$$= -\frac{\mu_0 i}{4\pi x_0} \wedge \cdot 2 = -\frac{\mu_0 i}{2\pi x_0} \hat{z}$$

 $B = \frac{\mu_0 i}{2\pi R}$ 通同向电流的导线相互吸引

$$F = BIL$$

$$E = \frac{\mu_0 i_1 i_2}{2\pi d}$$

单位长度受力

6

Thursday

农历五月初一

星期四

安培环路定理

闭合回路对磁场求积分

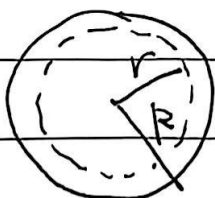
□ $\oint \vec{E} \cdot d\vec{l} = 0$ (静电场为保守场) $\oint \vec{B} \cdot d\vec{l} = \mu_0 i = \mu_0 \sum i_{enc}$ □ i 与积分回路方向一致时为正□ i_{enc} 为在以闭合回路为边界的任一面积内穿过的电流总量 $\oint \vec{B} \cdot d\vec{l} = \mu_0 \int \vec{j} \cdot d\vec{s}$

定理成立条件: 稳恒电流 (空间电流分布不变)

① 无限长直导线: $\oint \vec{B} \cdot d\vec{l} = \oint B dl = B 2\pi r = \mu_0 i$

$$B = \frac{\mu_0 i}{2\pi r}$$

② 半无限长直导线: 电流均匀分布



$$B \cdot 2\pi r = \mu_0 \left(\frac{r^2}{R^2} \right) i$$

$$B = \frac{\mu_0 i}{2\pi R} \frac{r^2}{R^2} = \frac{\mu_0}{2} j r$$

$$\vec{B} = \frac{\mu_0}{2} \vec{j} \times \vec{r}$$

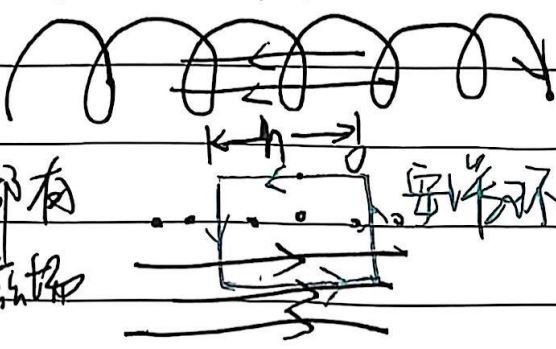
★在距离电流环的情况下, $B = \frac{\mu_0}{4\pi} \cdot \frac{2\mu}{r^3}$ (在极轴上).
 $= \frac{\mu_0}{2\pi} \frac{\mu}{r^3}$ 6月

$\frac{dB}{dr} = \frac{\mu_0}{4\pi} \frac{4Ide}{r^2}$

$dB = 7 \frac{\mu_0 I de}{4\pi r^2}$ Friday 农历五月初三 星期五
 年龄不是用来记数的, 你可以用它来做阶段性的反省, 但“什么年龄做什么事”这个事情根本就不存在, $= \frac{\mu_0 I S}{2\pi r^3}$

- ③ 板中均匀通有电流 J 为线电流密度 $\downarrow B$
- 取矩形安培回路 $2Bl = \mu_0 (J \cdot l)$
- $B = \frac{\mu_0 J}{2}$ 与到板反间距无关

④ 无限长螺线管



附加都有 螺线管 安培环路 $Bh = N h i \cdot \mu_0$
 $B = \mu_0 (\frac{N}{h}) i = \mu_0 n i$
 solenoid 螺线管 单位长度匝数 n

8 Saturday 农历五月初三 星期六

⑤ 长螺线管内部

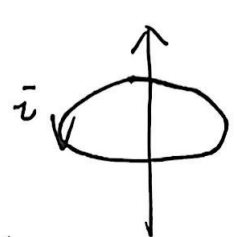
$2\pi r B = \mu_0 N i$
 $B = \frac{\mu_0 N i}{2\pi r}$

$\nabla \cdot B = 0$

$\nabla \times B = \mu_0 J$ (Stokes 公式) 安培环路定理微分形式

$\oint B \cdot dS = 0$ 磁场的高斯定理

Lecture 10 Magnetic Properties of Materials



M_s 与电流环形成的场类似
 magnetic dipole

- magnetic dipole's origins:
- ① 带电流的线圈
 - ② 永磁体
 - ③ a rotating sphere of charge 自转的一团电荷 e.g. 地球
 - ④ a subatomic particle 带电粒子转动 (质子, 中子) 自旋 spin

9th

Sunday
农历五月初四
星期日

自旋↑↓ 与外磁场平行或反平行

轨道磁矩与B的夹角 $\tau = -\mu_B \sin\theta = -\frac{\partial}{\partial\theta}(-\mu_B \cos\theta) = -\frac{\partial}{\partial\theta}(-\mu \cdot \vec{B})$
 磁偶极子与外场同时能量最低。磁偶极子在磁场中具有的能量
 对于自旋产生的磁矩，只有两种状态↑↓。 $\pm \frac{1}{2}$ ← 磁矩大小
 约化普朗克常数 = $\frac{h}{2\pi}$ → 普朗克常数

轨道磁矩 可连续变化

顺磁 每个原子都有单独磁矩(有未成对电子) 原子和原子上的磁矩的相互作用若很弱，不能形成有序排列，整体无磁矩 (paramagnetism) 若较强，磁矩将沿B排列达到最低能量

10

Monday
农历五月初五
端午节星期一

没有武艺傍身，就不要出来行走江湖。

磁矩与磁场的相互作用

(被极化)

铁磁 Ferromagnetism

每个原子有自己的磁矩，且磁矩间有较强的相互作用

能量 $U = -\sum_i \vec{\mu}_i \cdot \vec{B}$

前半部分为海森堡模型

J>0 铁磁 同向排列 降低能量

J<0 反铁磁 反向排列

铁磁在温度很高时 无序排列 (向降低能量 大于排列降低能量)

本周计划 TOP PRIORITIES

居里定律 $M = C \frac{B}{T}$ T → 温度

在顺磁情况下有最大极化点 $N \propto e^{-\mu/k_B T}$

$N_+ = N \frac{e^{\beta \mu_B}}{e^{\beta \mu_B} + e^{-\beta \mu_B}}$ $\beta = \frac{1}{k_B T}$

$N_- = N \frac{e^{-\beta \mu_B}}{e^{\beta \mu_B} + e^{-\beta \mu_B}}$

$M = (N_+ - N_-) \mu = N \frac{\mu_B}{k_B T}$

抗磁 Diamagnetism 原子无未成对电子 电子绕行的电子形成轨道磁矩，形成

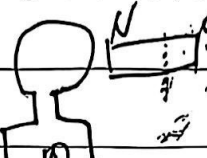
抵消外磁场的场。(排斥作用)

11

Tuesday
农历五月初六
星期二

Lecture 11 Faraday's Law of Induction

变化的电场产生磁场 变化的磁场产生电场

□ 法拉第电磁感应定律.  产生的电流为

□ 实验 -  感生电流

□ 实验 = 用通电线圈代替磁铁. 闭合开关及断开瞬间

$\times \times \times \times \vec{B}$ 产生感应电动势. 产生感生电流

$\times \times \times \times \vec{B}$ 

$\times \times \times \times$ 载流子为电子. $\downarrow F$

$$\mathcal{E} = - \frac{d\Phi_B}{dt}$$

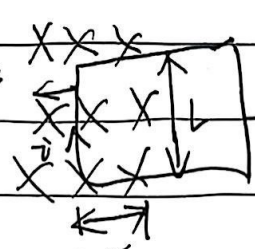
$$\oint \vec{E} \cdot d\vec{l} = - \frac{d}{dt} \int \vec{B} \cdot d\vec{S} \quad \text{微分形式: } \nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

对于N匝 $\mathcal{E} = -N \frac{d\Phi_B}{dt}$

楞次定律: 加负号的原因 (能量守恒)

12

Wednesday
农历五月初七
星期三

$\Phi = BLx$

 $\mathcal{E} = BLv$
 $F = BIL$
 $P_{\text{外力}} = F \cdot v = \frac{B^2 L^2 v^2}{R}$

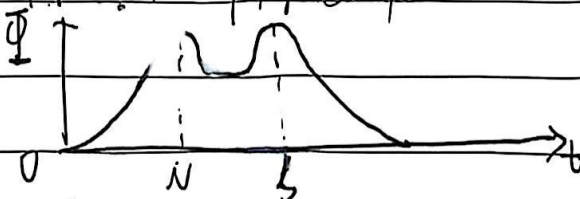
$$i = \frac{\mathcal{E}}{R} = \frac{BLv}{R}$$

$$P_L = i^2 R = \frac{B^2 L^2 v^2}{R}$$

$$P_{\text{外力}} = F \cdot v = \frac{B^2 L^2 v^2}{R}$$

滑轮系统不做功. 外力做功

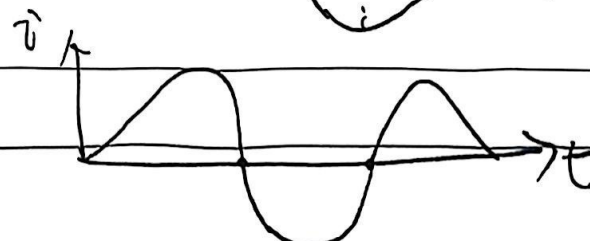
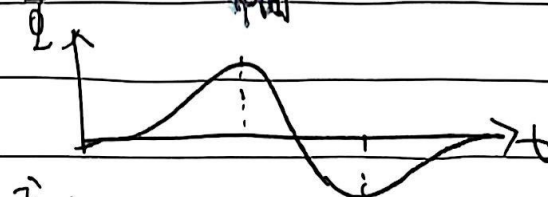
□ $S \rightarrow N \rightarrow$



 N极接近
到环上时
Φ最大



磁单极子 



13

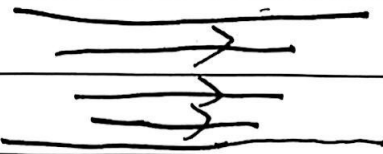
Thursday
农历五月初八
星期四

若无闭合回路则无感应电动势。
即使无闭合回路，只要有磁场变化便会产生电场

□ 螺线管可产生圆形均匀磁场

□ Lecture 12 Inductors and Inductance

□ $B = \mu_0 n I$. n 为单位长度匝数



密绕螺线管

$$\Phi_B = BA = \mu_0 n I A$$

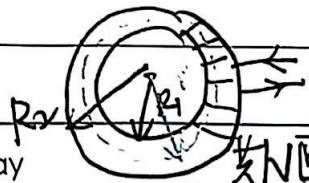
$\frac{\Phi}{i} = \mu_0 n A$ 右侧与外加电流无关
对长度为 l 的螺线管，共 $n l$ 匝

$$\Phi_{\text{总}} = \mu_0 n^2 l A i$$

电感/自感系数 $= \frac{N \Phi_B}{i}$
螺线环

N 为匝数 l 单位 H 亨利

假设截面为矩形



以 i 作环路

14

Friday
农历五月初九
星期五

咀嚼和饱胀的欲望之上，还有更高级的欲望叫做理性和自律。

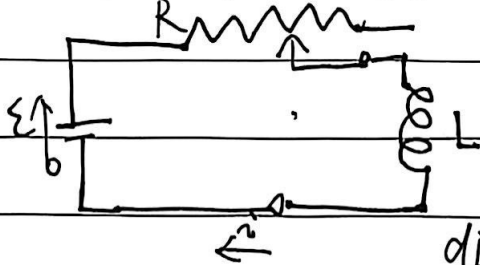
$$\oint \vec{B} \cdot d\vec{r} = B \cdot 2\pi r = \mu_0 \sum i = \mu_0 N i$$

□ $B = \frac{\mu_0 N i}{2\pi r}$ $\Phi_B = \int \vec{B} \cdot d\vec{S} = \int_{R_1}^{R_2} \frac{\mu_0 N i}{2\pi r} h dr = \frac{\mu_0 N i h}{2\pi} \ln \frac{R_2}{R_1}$

□ $L = \frac{N \Phi_B}{i} = \frac{\mu_0 N^2 h}{2\pi} \ln \frac{R_2}{R_1}$ 对同一螺线环为常量

□ $\Phi = L i$

自感 $\mathcal{E}_L = -\frac{d\Phi}{dt} = -L \frac{di}{dt}$ 由于自身磁通量变化产生的感生电动势。



$$\mathcal{E} - iR - L \frac{di}{dt} = 0 \quad (1)$$

$$\frac{\mathcal{E}}{R} - \frac{i}{R} = \frac{di}{dt}$$

L 具有时间的量纲。

$$\frac{di}{i - i_0} = -\frac{dt}{\tau} \quad \tau = \frac{L}{R} = \frac{1}{R} \frac{d\Phi}{di} \quad \frac{L}{R} = \tau$$

由(1)乘 i : $i\mathcal{E} - i^2 R - L i \frac{di}{dt} = 0$

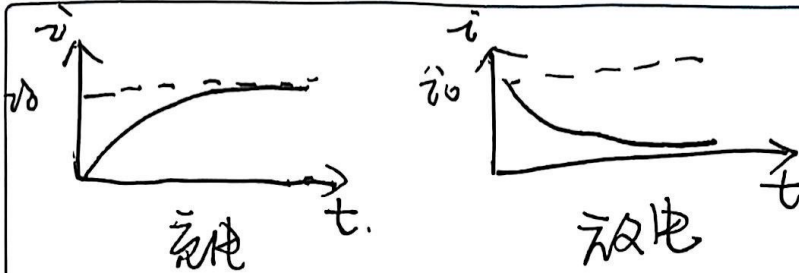
$\int L i \frac{di}{dt} \cdot dt$ 为线圈储存能量
 $= \frac{1}{2} L i^2 = \mathcal{E}_B$

放电过程 $-\frac{d\mathcal{E}_B}{dt} = i^2 R \quad L \frac{di}{dt} + iR = 0 \quad \frac{di}{dt} + \frac{i}{\tau} = 0 \quad i = i_0 e^{-t/\tau}$

15

Saturday
农历五月初十
星期六

刚充电时视为断路, 充完后可视为短路



$$L = \mu_0 n^2 l A \quad \mathcal{E}_B = \frac{1}{2} L i^2 = \frac{1}{2} \mu_0 (\mu_0 n i)^2 A l \rightarrow V$$

对螺线管: $\mathcal{E}_B = \frac{1}{2} \mu_0 B^2$

对螺线管: $B = \frac{\mu_0 n i}{2\pi r} \quad W_B = \frac{\mu_0 n^2 i^2}{8\pi^2 r^2}$

$$\mathcal{E}_B = \int W_B dV = \int W_B 2\pi r h dr = \frac{\mu_0 n^2 i^2}{4\pi} h \ln \frac{R_2}{R_1} = \frac{1}{2} L i^2$$

$$L = \frac{\mu_0 n^2 h}{2\pi} \ln \frac{R_2}{R_1} \quad \text{第2个线圈的磁通}$$

16

Sunday
农历五月十一
父亲节星期日

$$B \propto i \quad \mathcal{E}_B \quad N_2 \Phi_{21} = M_{21} i_1 \quad M_{12} = M_{21} = M$$

Mutual induction

$$\mathcal{E}_1 = - \frac{d(N_1 \Phi_{12})}{dt} = - M \frac{di_2}{dt} \quad \mathcal{E}_2 = - \frac{d(N_2 \Phi_{21})}{dt} = - M \frac{di_1}{dt}$$

$$N_2 \Phi_{21} = M i_1$$

$$1) \text{ if } \mathcal{E}_2 = 0 \quad \mathcal{E}_1 = 0 \rightarrow V_1, i_1$$

$$2) \text{ if } \mathcal{E}_1 = V_1 \quad \mathcal{E}_2 = 0 \rightarrow V_2, i_2$$

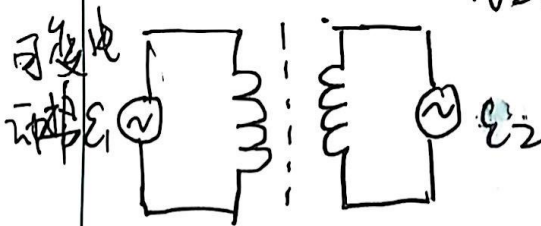
$$3) \text{ if } \mathcal{E}_1 = 0 \quad \mathcal{E}_2 = 0 \rightarrow V_2, i_2$$

$$\mathcal{E}_2 = V_2 \quad \mathcal{E}_1 = 0 \rightarrow V_1, i_1$$

$$1) W_1 = \frac{1}{2} L_1 i_1^2$$

$$W_2 = \frac{1}{2} L_2 i_2^2 + \int i_1 M \frac{di_2}{dt} dt = \frac{1}{2} L_2 i_2^2 + M i_1 i_2$$

$$W_1 + W_2 = W_1' + W_2' \quad M_{12} = M_{21} = M$$



17

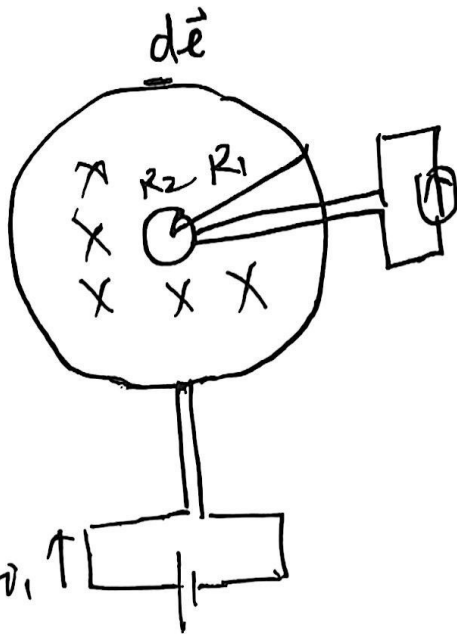
Monday
农历五月十二
星期一

哪有不经过彩排和推敲的好戏？偶然的精彩是运气，经常性的精彩一定是运维的结果。

本周计划 TOP PRIORITIES

W25

- ☐
- ☐
- ☐
- ☐
- ☐
- ☐
- ☐
- ☐



由于 R_2 较小, 可将 中心处磁

场视为 R_2 磁偶极

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{l} \times \vec{r}}{r^2}$$

$$\oint \vec{B} = \int \frac{\mu_0}{4\pi} \frac{i d\vec{l} \times \vec{r}}{r^2} = \frac{\mu_0 i}{2R_1}$$

$$B = \frac{\mu_0 M i}{2R_1}$$

$$\Phi_2 = B \pi R_2^2 = \frac{\mu_0 M i \pi R_2^2}{2R_1}$$

$$M = \frac{\Phi_2}{i} = \frac{\mu_0 M i \pi R_2^2}{2R_1}$$

磁偶极子产生的磁偶极矩

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{1}{r^3} [3(\vec{m} \cdot \vec{r})\vec{r} - m^2]$$

18

Tuesday
农历五月十三
星期二

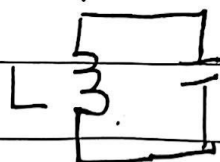
若考虑 L_2 对 L_1 的磁矩可用磁偶极子

$$\vec{B} = -\frac{\mu_0}{4\pi} \frac{m}{r^3}$$

$$\Phi_{\text{外}} = \int_{R_1}^{\infty} -\frac{\mu_0}{4\pi} \frac{is}{r^3} 2\pi r dr \quad \Phi_{\text{内}} = -\Phi_{\text{外}}$$

Lecture 13 Alternating Current Circuits.

LC 振荡回路 $T_C = \frac{1}{\omega_C}$ $T_C = \frac{L}{R}$



$$T_{LC} = \sqrt{LC}$$

理想 = 无 R. 忽略电磁波耗散

$$U_E + U_B = \text{const.} \quad \frac{d}{dt}(U_B + U_E) = 0$$

$$U_B = \frac{1}{2} L i^2 \quad U_E = \frac{q^2}{2C} \quad L i \frac{di}{dt} + \frac{q}{C} \frac{dq}{dt} = 0$$

$$L \frac{d^2 q}{dt^2} + \frac{q}{C} = 0 \quad \ddot{q} + \frac{q}{LC} = 0$$

$$q = Q \cos(\omega t + \phi) \quad i = \frac{dq}{dt} = -\omega Q \sin(\omega t + \phi)$$

$$V = \frac{q}{C} = \frac{Q}{C} \cos(\omega t + \phi)$$

V 最小, i 最大. 相差 $\frac{\pi}{2}$.

i 快 $\frac{1}{\pi}$

2024

$$\frac{H \cos^2(\omega t + \phi)}{2}$$

6月

19

Wednesday
农历五月十四
星期三

$$U_E = \frac{q^2}{2e} = \frac{Q^2}{2e} \cos^2(\omega t + \phi)$$

$$U_B = \frac{1}{2} L i^2 = \frac{1}{2} L Q^2 \omega^2 \sin^2(\omega t + \phi) \sim \frac{1 - \cos 2(\omega t + \phi)}{2}$$

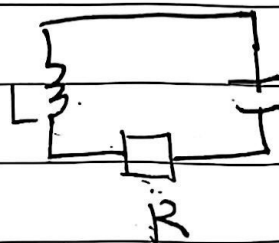
$$= \frac{Q^2}{2C} \sin^2(\omega t + \phi)$$

☐

☐

☐

实际:



RC: 放电 $q = Q e^{-t/\tau} = Q e^{i\omega t}$ $\omega = \frac{1}{\tau}$

LC: $q = Q \cos(\omega t + \phi) = \text{Re}(Q e^{i\omega t} e^{i\phi})$

通式 $q = \text{Re}(\tilde{Q} e^{i\tilde{\omega} t})$ $\tilde{Q}, \tilde{\omega}$ 为复数

$$\frac{d}{dt}(U_E + U_B) = i^2 R$$

$$L i \frac{di}{dt} + \frac{q}{C} \frac{dq}{dt} + i^2 R = 0$$

$$L \frac{dq}{dt} + \frac{q}{C} + iR = 0 \quad i = \frac{dq}{dt}$$

$$L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = 0$$

$$\Leftrightarrow L \frac{d^2 \tilde{q}}{dt^2} + \tilde{q} + \frac{d\tilde{q}}{dt} R = 0$$

$$-L\omega^2 \tilde{Q} e^{i\omega t} + R i \omega \tilde{Q} e^{i\omega t} + \frac{\tilde{Q}}{C} e^{i\omega t} = 0$$

$$\Rightarrow L\omega^2 - iR\omega - \frac{1}{C} = 0 \quad \frac{R}{L} = 2\omega_1$$

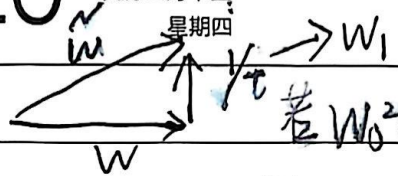
20

Thursday
农历五月十五
星期四

$$L\omega^2 - \frac{1}{C} \tilde{\omega}^2 = 0$$

$$\tilde{\omega}^2 - i \cdot 2\omega_1 \tilde{\omega} - \omega_0^2 = 0 \quad \tau = \frac{1}{\omega_1} = \frac{2L}{R}$$

$$\tilde{\omega} = i\omega_1 \pm \sqrt{\omega_0^2 - \omega_1^2}$$



若 $\omega_0^2 = \omega_1^2$

$$\frac{1}{C} > (\frac{R}{2L})^2 \quad R < \frac{2L}{\sqrt{LC}} \quad \text{令 } \omega = \pm \sqrt{\omega_0^2 - \omega_1^2}$$

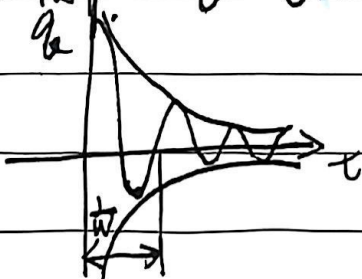
$$\text{代入 } q = \text{Re}\{\tilde{Q} e^{i\omega t}\}$$

$$e^{-\omega_1 t} e^{\pm i\omega t} \quad \cos(\omega t)$$

ω_1 提供指数衰减部分

提供振荡部分

解: $q = Q e^{-t/\tau} \cos(\omega t + \phi)$



若 $R > \frac{2L}{\sqrt{LC}}$ 两个指数衰减叠加

$$\tilde{Q} e^{-\omega_1 t}$$

$$\omega_{\pm} = i(\omega_1 \pm \sqrt{\omega_1^2 - \omega_0^2})$$

$\tau_{\pm} = \frac{1}{\omega_{\pm}}$ 代表衰减快慢

总体衰减时间大于 $\frac{2L}{R}$

若 $\frac{1}{C} = \omega_0^2$ 即 $\omega_1 = \omega_0 \quad \sqrt{\omega_0^2 - \omega_1^2} = 0 \quad \tau_+ = \tau_- = \frac{2L}{R}$

critically damped 回到平衡位置所需时间最短

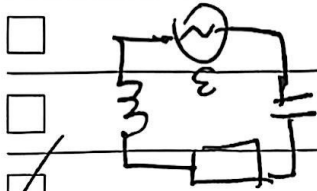
AC电路

21

Friday

农历五月十六
夏至 星期五

人只有见识过若干种活法，对自己能拥有的活法才具备了想象力。



$$\varepsilon = \varepsilon_0 \cos(\omega t + \phi) \quad i = \operatorname{Re}\left\{\frac{\tilde{\varepsilon}}{Z}\right\} = \operatorname{Re}\left\{\frac{\tilde{\varepsilon}_0}{Z} e^{i\omega t}\right\}$$

$$\tilde{\varepsilon} = \tilde{\varepsilon}_0 e^{i\omega t} \quad \tilde{\varepsilon}_0 = \varepsilon_0 e^{i\phi}$$

$$i = \operatorname{Re}\left\{\tilde{i}_0 e^{i\omega t}\right\} \quad \tilde{i}_0 = \frac{\tilde{\varepsilon}_0}{Z} \quad \text{对于 } \bigcirc \text{ 和 } \square$$

对于 $\bigcirc$ $\tilde{V} = \varepsilon = \frac{q}{C} \quad \tilde{i} = \frac{dq}{dt} = \frac{d(\operatorname{Re}\{\tilde{\varepsilon}\})}{dt} = C \frac{d}{dt}(\operatorname{Re}\{\tilde{\varepsilon}_0 e^{i\omega t}\})$
 $= \operatorname{Re}\{iC\tilde{\varepsilon}_0 \omega e^{i\omega t}\} = \operatorname{Re}\{\tilde{i}_0 e^{i\omega t}\}$

$$\tilde{i}_0 = i\omega C \tilde{\varepsilon}_0$$

$$R = Z_R = R \quad C: Z_C = \frac{1}{i\omega C} \quad R: \tilde{i}_0 = \frac{\tilde{\varepsilon}_0}{Z_R} \quad C: \tilde{i}_0 = \frac{\tilde{\varepsilon}_0}{Z_C}$$

对于 $\square$ $L \frac{di}{dt} = \varepsilon \quad Z_L = i\omega L$

对于 L-C 串联电路阻抗处理 Z 为阻抗

$$\tilde{i}_0 = \frac{\tilde{\varepsilon}_0}{Z} = \frac{\tilde{\varepsilon}_0}{R + i(\omega L - \frac{1}{\omega C})} = \frac{\tilde{\varepsilon}_0 [R - i(\omega L - \frac{1}{\omega C})]}{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

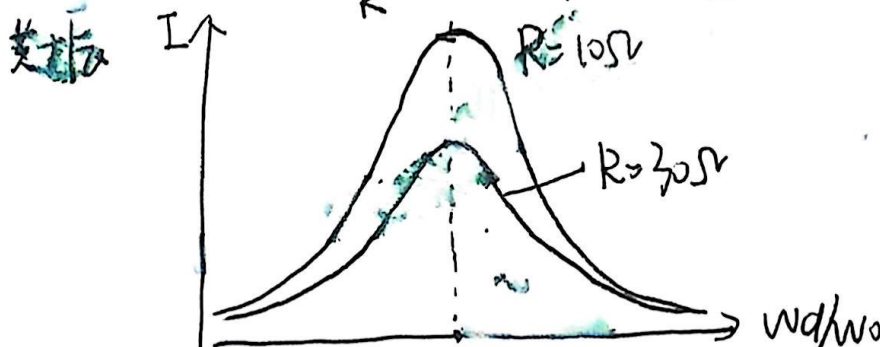
22

Saturday
农历五月十七
星期六

若 $\tilde{i}_0 = \frac{\tilde{\varepsilon}_0}{A}$ A 为实数，则无相位差

相位差 $\tan \phi = \frac{\omega L - 1/(\omega C)}{R}$

若 $\omega L = \frac{1}{\omega C} \Rightarrow \frac{\tilde{\varepsilon}_0}{R}$ 最大振幅



Lecture 14 Maxwell's Equations and EM Waves

$\nabla \cdot (\nabla \times \vec{A}) = 0$ 矢量场的旋度的散度为0

$$\oint \vec{E} \cdot d\vec{s} = \frac{1}{\varepsilon_0} \int \rho dV \quad \oint \vec{B} \cdot d\vec{s} = 0 \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \nabla \cdot \vec{B} = 0$$

$$\oint \vec{E} \cdot d\vec{r} = -\frac{d}{dt} \oint \vec{B} \cdot d\vec{s} \quad \oint \vec{B} \cdot d\vec{r} = \mu_0 \oint \vec{j} \cdot d\vec{s} \quad \nabla \times \vec{B} = \mu_0 \vec{j}$$

23

Sunday
农历五月十八
星期日

$$\nabla \cdot (\nabla \times \vec{E}) = \nabla \cdot \left(-\frac{\partial \vec{B}}{\partial t} \right) = -\frac{\partial}{\partial t} (\nabla \cdot \vec{B}) = 0$$

$$\nabla \cdot (\nabla \times \vec{B}) = \nabla \cdot (\mu_0 \vec{J}) = \mu_0 (\nabla \cdot \vec{J})$$

由电荷的连续性方程 $\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$

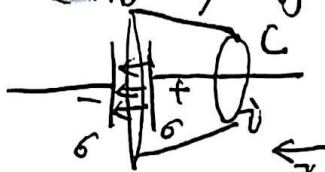
$$\mu_0 (\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t}) = 0$$

$$\oint \vec{J} \cdot d\vec{S} + \int \frac{\partial \rho}{\partial t} dV = 0$$

$$\oint \vec{J} \cdot d\vec{S} + \frac{d}{dt} \int \rho dV = 0$$

$$\mu_0 (\nabla \cdot \vec{J} + \frac{\partial}{\partial t} \epsilon_0 \nabla \cdot \vec{E}) = \mu_0 (\nabla \cdot \vec{J} + \epsilon_0 \nabla \cdot \frac{\partial \vec{E}}{\partial t})$$

$$\therefore \nabla \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right) \Rightarrow \int \vec{B} \cdot d\vec{l} = \mu_0 \int \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right) \cdot d\vec{S}$$



$$\vec{E} = \frac{\sigma}{\epsilon_0} \hat{x} \quad \frac{\partial \vec{E}}{\partial t} = \frac{1}{\epsilon_0} \frac{\partial \sigma}{\partial t} \hat{x}$$

位移电流 $\vec{J}_d$

$$\int \epsilon_0 \frac{\partial \vec{E}}{\partial t} \cdot d\vec{S} = \int \frac{\partial \sigma}{\partial t} \hat{x} \cdot d\vec{S} = \frac{\partial \sigma}{\partial t} \cdot A = \frac{\partial (QA)}{\partial t}$$

真空中 $\nabla \cdot \vec{E} = \rho / \epsilon_0$ $\nabla \cdot \vec{B} = 0$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$= \frac{\partial q}{\partial t} = I$$

24

Monday
农历五月十九
星期一

日月精华不止在于人，更在于星辰大海。

$$\vec{E} = E_0 \cos(\omega t - \vec{k} \cdot \vec{r} + \phi)$$

波矢 $\vec{k}$ 传播方向

本周计划 TOP PRIORITIES

W26

☐ $\nabla \times (\nabla \times \vec{E}) = \nabla (\nabla \cdot \vec{E}) - (\nabla \cdot \nabla) \vec{E}$

☐ $\nabla \times \left(-\frac{\partial \vec{B}}{\partial t} \right) = -\nabla^2 \vec{E}$

☐ $-\nabla^2 \vec{E} = -\frac{\partial}{\partial t} (\nabla \times \vec{B})$

☐ $= -\mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \vec{E}$

☐ $\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \vec{E} \quad \nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \vec{B}$

☐ 若一维 $\frac{\partial^2}{\partial x^2} \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \vec{E}$

☐ 波动方程 $\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}$

其通解为 $f = f_1(x-vt) + f_2(x+vt)$

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c$$

光是一种电磁波

$$|\vec{k}| = \omega$$

沿 x 方向传播

$$\vec{E} = E_0 \cos(\omega t - kx + \phi)$$

$$\vec{B} = B_0 \cos(\omega t - kx + \phi)$$

$$\nabla \times \vec{E} = \frac{\partial E_y}{\partial x} \hat{z} - \frac{\partial E_z}{\partial x} \hat{y}$$

$$= -E_0 y (k) \sin(\omega t - kx + \phi) \hat{z} - E_0 z k \sin(\omega t - kx + \phi) \hat{y}$$

$$= k \sin(\omega t - kx + \phi) (E_0 y \hat{z} - E_0 z \hat{y})$$

$$= -\frac{\partial \vec{B}}{\partial t} = B_0 \omega \sin(\omega t - kx + \phi)$$

有 $\phi = \phi'$

2024

 $\frac{1}{c}$

June

25

Tuesday
农历五月二十
星期二

$$\frac{k}{\omega}(\epsilon_0 \omega^2 \hat{e} - \epsilon_0 \omega^2 \hat{e}) = \vec{B}_0 \quad \text{无入分量, 即与传播方向} \perp$$

$$\vec{B} \perp \vec{k} \quad \vec{E} \perp \vec{k} \quad \vec{E} \perp \vec{B}$$

☐

$$\vec{E}_0 = \epsilon_0 \omega^2 \hat{y} + \epsilon_0 \omega^2 \hat{z} \quad \vec{E}_0 \cdot \vec{B}_0 = 0$$

☐

$$\vec{B} = \frac{1}{c}(\hat{k} \times \vec{E})$$

 $(\hat{k}, \vec{E}, \vec{B})$ 右手螺旋系☐

电磁波为横波

$$W_E = \frac{1}{2} \epsilon_0 E^2$$

$$W_B = \frac{1}{2} \mu_0 B^2 \quad \text{在电磁波中 } W_E = W_B$$

$$\text{取模 } B = \frac{E}{c}$$

电磁波运动的主要为电场

$$W = W_E + W_B$$

$$S = (W_E + W_B)C$$

能流密度 $\vec{S}$ $\nabla \cdot \vec{S} + \frac{\partial W}{\partial t} = 0$ 能量守恒

$$(\text{单位 } \text{Watt/m}^2) \nabla \cdot \vec{S} = -\frac{\partial W}{\partial t} = -\frac{\partial}{\partial t}(W_E + W_B) = -\frac{\partial}{\partial t} \left[\frac{1}{2} \epsilon_0 (\vec{E} \cdot \vec{E}) + \frac{1}{2} \mu_0 (\vec{B} \cdot \vec{B}) \right]$$

$$= -\left(\epsilon_0 \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} + \frac{1}{\mu_0} \vec{B} \cdot \frac{\partial \vec{B}}{\partial t} \right)$$

$$\mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} = \nabla \times \vec{B} \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

26

Wednesday
农历五月廿一
星期三

$$= \frac{1}{\mu_0} [\vec{B} \cdot (\nabla \times \vec{E}) - \vec{E} \cdot (\nabla \times \vec{B})] = \frac{1}{\mu_0} \nabla \cdot (\vec{E} \times \vec{B})$$

$$\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$$

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} = c \hat{k} \frac{B_0^2}{2} \cos^2(\omega t - kx + \phi)$$

代表单位面积能流

☐

Lecturer 5 Light as Electromagnetic Waves

$$\vec{E} = \vec{E}_0 \cos(kx - \omega t)$$

$$\text{Wave number } k = \frac{2\pi}{\lambda}$$

$$\omega = ck = c \frac{2\pi}{\lambda}$$

dispersion relation $\omega = ck$

$$\text{Gauss law } \vec{k} \cdot \vec{E} = 0$$

$$\vec{k} \cdot \vec{B} = 0$$

$$\vec{k} \times \vec{E} = \omega \vec{B}$$

$$-\vec{k} \times \vec{B} = \mu_0 \epsilon_0 \omega \vec{E}$$

$$\text{in 3D } \vec{E} = \vec{E}_0 \cos(\vec{k} \cdot \vec{r} - \omega t)$$

$$\omega = c|\vec{k}|$$

$$\vec{k} \cdot \vec{r} = \text{constant} \quad \text{wavefront}$$

$$\text{if in a medium } \omega = \left(\frac{c}{n}\right) \frac{2\pi}{\lambda}$$

27

Thursday
农历五月廿二
星期四

$$\vec{k} = \frac{2\pi}{\lambda} \vec{r}. \quad |\vec{k}| = 1.$$

wavefront 不一定为平面



rays are perpendicular to the wavefront



$$v = \frac{1}{\sqrt{\epsilon_r \mu_r} \sqrt{\epsilon_0 \mu_0}} = \frac{c}{n}. \quad n = \sqrt{\epsilon_r \mu_r} \text{ 为折射率. } n > 1$$



$$w = vk = \frac{ck}{n} \quad \text{介质中传播 } f \text{ 不变} \quad \epsilon_r(w), \mu_r(w)$$

$$w = c \frac{2\pi}{\lambda} = v \frac{2\pi}{\lambda} \quad v = \frac{c}{n} \quad \lambda' = \frac{\lambda}{n} \rightarrow n(w).$$

Snell's law $n_2 \sin \theta_2 = n_1 \sin \theta_1$ 折射

光疏到光密. 折射角变小

$$n_1 < n_2$$

红光折射率小于蓝光

Fermat's Principle 对均匀介质, 光作直线传播.

两点间光的传播沿所需时间最短的路径

mirage 海市蜃楼

快 n_1

path

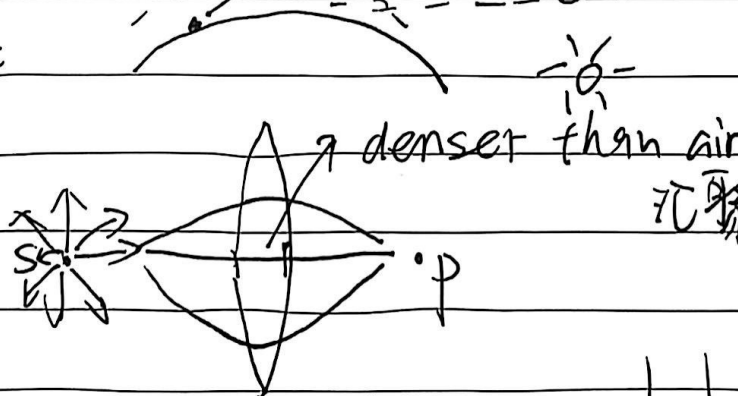
28

Friday
农历五月廿三
星期五

内心深处还是觉得不够重要, 才会做不到.

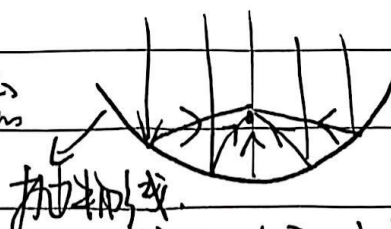


日落



汇聚于 P 点.

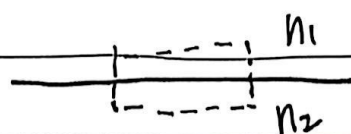
satellite dish 卫星接收器



Huygens' Principle 惠更斯原理 球形波面上每一点(面源)都是一个次级球面波的子波源.

The electromagnetic Approach

$$\oint \vec{E} \cdot d\vec{l} = - \frac{d\Phi_B}{dt} = 0.$$



29

Saturday
农历五月廿四
星期六

$\oint \vec{E} \cdot d\vec{s} = E_{\parallel}^{\text{up}} d - E_{\parallel}^{\text{down}} d \Rightarrow$ 边界条件.

$$\hat{n} \times (\vec{E}_t + \vec{E}_r) = \hat{n} \times (\vec{E}_t)$$

$$\hat{n} \times \vec{E}_o \cos(\vec{k}_i \cdot \vec{r} - \omega t) + \hat{n} \times \vec{E}_r \cos(\vec{k}_r \cdot \vec{r} - \omega t + \phi_r) = \hat{n} \times \vec{E}_t \cos(\vec{k}_t \cdot \vec{r} - \omega t + \phi_t)$$

$$\therefore \vec{k}_i \cdot \vec{r} = \vec{k}_r \cdot \vec{r} + \phi_r = \vec{k}_t \cdot \vec{r} + \phi_t$$

$$(\vec{k}_i - \vec{k}_r) \cdot \vec{r}_{y=c} = \phi_r \quad (\vec{k}_i - \vec{k}_r) \cdot (\vec{r}_1 - \vec{r}_2) = 0 \Rightarrow \hat{n} \times (\vec{k}_i - \vec{k}_r) = 0$$

$$k_i \sin \theta_i = k_r \sin \theta_r \Rightarrow \theta_i = \theta_r$$

$$(\vec{k}_i - \vec{k}_r) \cdot (\vec{r}_1 - \vec{r}_2) = 0 \Rightarrow \hat{n} \times (\vec{k}_i - \vec{k}_r)$$

不同坐标系下 $\vec{E} \cdot \vec{B} = C$. $\vec{E} \cdot \vec{B}$ 为能量.

Lecture 16 Geometrical Optics.

Concave mirror
凹面镜

视场更小

影像在镜后更远, 即 $|I|$ 更大.

图像高度增大

30

Sunday
农历五月廿五
星期日

凸面镜 convex mirror

物距 p . 像距 i . 对凸面镜 当 $p \rightarrow \infty$ 时.



F 到镜面距离即焦距.

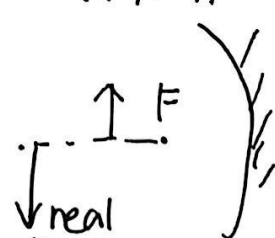
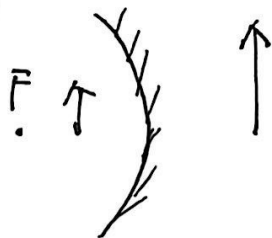
平行轴近似.

↑

平行轴近似: 在接近中心轴区域可用圆弧代替.

(接近轴由补线)

$$f = \frac{r}{2}$$



$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$$

适用于凹面镜与凸面镜

$p > 2f$ 倒立缩小实像

$p = 2f$ 等大

$f < p < 2f$ 放大

实像 $i > 0$

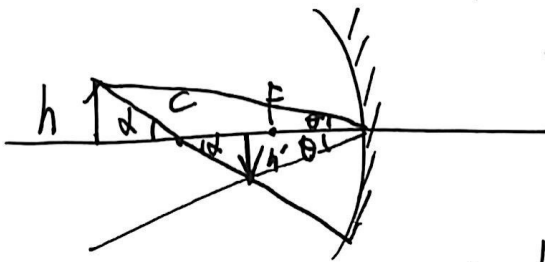
虚像 $i < 0$

本月总结

对凹面镜成像作图 ①平行光反射后过焦点
②过焦点反射后平行 ③C F 过C点反射后仍过C点 (2倍f)



④相同点看到物体一样大。



$$\tan \alpha = \frac{h}{p-2f} = \frac{h'}{2f-i} \quad \frac{h'}{h} = \frac{2f-i}{p-2f}$$

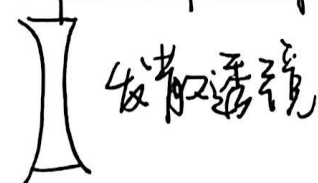
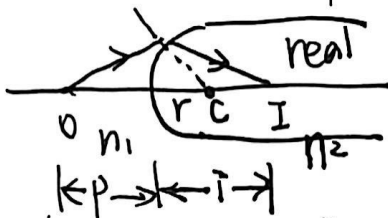
$$\tan \theta = \frac{h}{p} = \frac{h'}{i} \quad \frac{h'}{h} = \frac{i}{p}$$

$$m = \frac{h'}{h} = \pm \frac{i}{p} \quad \text{符号取决于像的虚实}$$

凸面折射与物体同侧成虚像, 异侧成实像
凸面折射时曲率为正, 凹面折射时曲率为负。

当光与中心轴夹角较小时 $\sin \theta \sim \theta$

$$\frac{n_1}{p} + \frac{n_2}{i} = \frac{n_2 - n_1}{r}$$



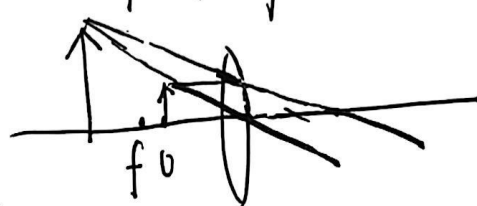
Thin lenses 薄透镜 使平行于中心轴的光线汇聚的透镜称为会聚透镜。

透镜中最厚的部分相对于p和i以及两面的曲率半径r1, r2均薄

lens maker's equation $\frac{1}{f} = (n-1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$

对薄透镜有 $\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$

放+镜



将物体置于焦点内
但靠近焦点

肉眼焦距约 25 cm.

显微镜 物在物镜焦距外, 形成倒立放大实像, 像在目镜焦距内, 形成放大虚像

1

Monday

农历五月廿六
建党节星期一

如果你要真的想做成一件事，那么在最开始就不该提“平衡”两个字。

本周计划 TOP PRIORITIES

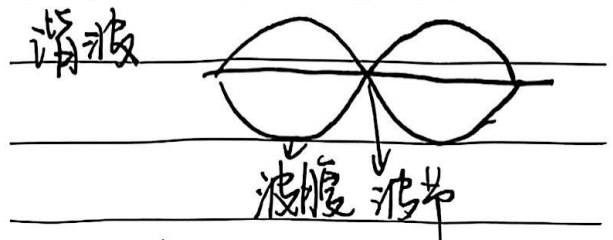
- ☐ $f_{ob} + s + f_{ey}$, 即两^{W27}镜头间距
- ☐ 为同一^{W27}透镜数为物镜焦距
- ☐ f_{ey} 为目镜焦距. s 为 length of microscope. 其值越大,
- ☐ 放大倍数越大.
- ☐ $M = m_1 m_2 \approx - \frac{f_{ob} + s}{f_{ob}} \cdot \frac{25\text{cm}}{f_{ey}}$
- ☐ $s \gg f_{ob}$ 时
- ☐ $M \approx - \frac{s}{f_{ob}} \frac{25\text{cm}}{f_{ey}}$

Lecture 17 Interference

Beats: 两频率相近的声波叠加.

$$\psi = \psi_1 + \psi_2 = A \cos \omega_1 t + A \cos \omega_2 t, \omega_1 \approx \omega_2$$

驻波: 两频率相同反向往传播的



$$\psi = \psi_1 + \psi_2 = A \cos(kx - \omega t) - A \cos(-kx - \omega t)$$

波叠加的干涉

① 同函数法

$$f, \lambda \text{ 相等 } E_1 = E_0 \cos(k_1 x - \omega t)$$

$$E_2 = E_0 \cos(k_2 x - \omega t)$$

$$\alpha_i = k X_i + \phi_i$$

$$E_1 + E_2 = (E_0 \cos \alpha_1 + E_0 \cos \alpha_2) \cos \omega t + (E_0 \sin \alpha_1 + E_0 \sin \alpha_2) \sin \omega t$$

$$E = E_0 \cos(\alpha - \omega t) = E_0 \cos \alpha \cos \omega t + E_0 \sin \alpha \sin \omega t$$

$$E^2 = E_0^2 + E_0^2 + 2E_0 E_0 \cos(\alpha_2 - \alpha_1) \text{ interference term}$$

$$[E^2]_{\text{avg}} = \frac{E_m^2}{2} \text{ The intensity of wave: } I = S_{\text{avg}} = \frac{E_0^2 \epsilon_0 c}{2}$$

$$\text{② 复数法 } \tilde{E} = E_0 e^{i(kx - \omega t)} \quad E = \text{Re}(\tilde{E}) = \frac{B_m^2}{2\mu_0} c$$

$$\text{Intensity } \tilde{E} \tilde{E}^* = E_0^2$$

$$\downarrow [E_0 e^{i(k_1 x - \omega t)} + E_0 e^{i(k_2 x - \omega t)}] [\dots]^*$$

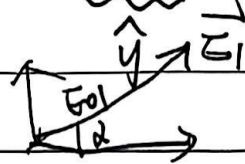
$$= E_0^2 + E_0^2 + E_0 E_0 [e^{i(k_1 - k_2)x} + e^{-i(k_1 - k_2)x}]$$

$$\text{③ phasor Addition } E_i = E_0 i \cos(k_i x - \omega t)$$

$$= E_0 i \cos \alpha_i \cos \omega t + E_0 i \sin \alpha_i \sin \omega t$$

$$\therefore \vec{E}_i = E_0 i \cos \alpha_i \hat{x} + E_0 i \sin \alpha_i \hat{y}$$

coherent 干涉 (f 同相干涉)

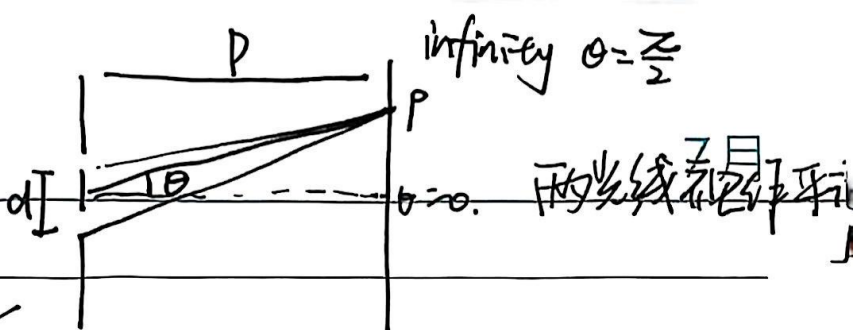


2024

3

Wednesday
农历五月廿八
星期三

双缝干涉



☐

$$d \sin \theta = d \sin \theta$$

☐

In-phase 同相

$$\Delta \Phi = k \Delta L = \frac{2\pi}{\lambda} d \sin \theta \quad k \Delta L = 2m\pi \quad m \in \mathbb{Z}$$

☐

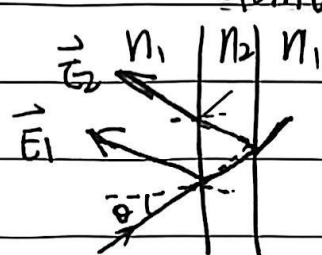
Out-of-phase 反相

$$k \Delta L = (2m+1)\pi$$

$$d \sin \theta = m\lambda$$

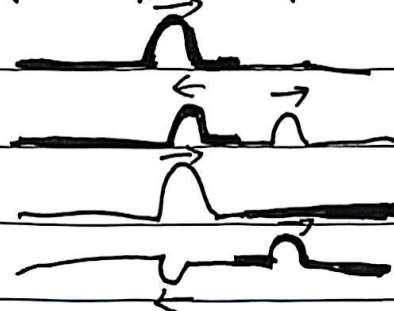
$$d \sin \theta = (m + \frac{1}{2})\lambda$$

薄膜干涉



$\vec{E}_1$ 与 $\vec{E}_2$ 相干

反射时光程外相位差



Reflection

phase shift

off lower index

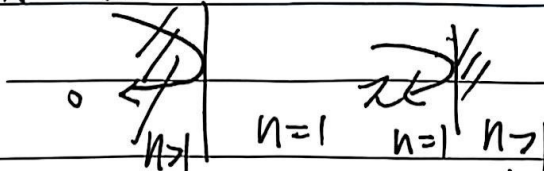
0

off higher index

π

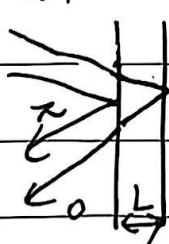
4

Thursday
农历五月廿九
星期四



☐

干涉薄膜干涉



综合考虑 ΔL 与相位差 π

☐

$$\theta = 0 \text{ 时 } \Delta L = 2L$$

☐

$$\Delta \Phi = k \Delta L + \pi = 2m\pi$$

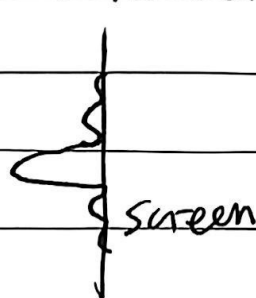
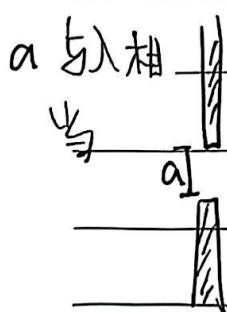
$$k \Delta L = (2m+1)\pi$$

maximum

$$\frac{2\pi n_2}{\lambda} 2L = (2m+1)\pi \quad 2L = (m + \frac{1}{2}) \frac{\lambda}{n_2}$$

out-of-phase. $2L = \frac{m\lambda}{n_2}$

Lecture 18 Diffraction 衍射



单缝衍射

夫琅禾费 vs 菲涅尔衍射

$$D > a/\lambda$$

菲涅尔衍射

远场衍射



2024

1st dark

5

Friday

农历五月三十

星期五

任何时候都应该明确方向,并核定时间及量化目标。这是一切事儿的道理。

Location of the Dark Fringes.

将狭缝分为2部分

P₁处 dark: r₁, r₂ 相互抵消.

$\Delta l = \frac{a}{2} \sin \theta = \frac{\lambda}{2}$ (2m+1)λ 中 m=0

1st dark fringe $\sin \theta = \frac{\lambda}{a}$ a很大时 均匀光强

2nd dark

a=λ时 全亮

2nd dark fringe $\sin \theta_{2nd} = \frac{2\lambda}{a}$ 将狭缝分为四区域

$\frac{a}{4} \sin \theta = \frac{\lambda}{2}$ r₁, r₂ 间, r₂, r₃ 间, r₃, r₄ 间 Δx 均为 $\frac{\lambda}{2}$

3rd $\sin \theta_{3rd} = \frac{3\lambda}{a}$ nth $\sin \theta_{nth} = \frac{n\lambda}{a}$

分区依据: W/50 为 1/3

求均匀分布: 复数法

6

Saturday

农历六月初一

星期六

$$\vec{E}_0 = \frac{E_m}{N} e^{-i\omega t} e^{ikr_1} + e^{ik(r_2-r_1)} + e^{ik(r_3-r_1)} + \dots + e^{ik(r_N-r_1)}$$

$$r_N - r_1 = \frac{(N-1)a}{N} \sin \theta$$

$$\frac{I(\theta)}{I_m} = \frac{E_0^2}{E_m^2} = \left[\frac{\sin \alpha}{\alpha} \right]^2 \quad \alpha = \frac{\pi a \sin \theta}{\lambda}$$

$$= \frac{E_m}{N} e^{-i\omega t} e^{ikr_1} \left(\frac{1 - e^{ika \sin \theta}}{1 - e^{ika \sin \theta / N}} \right)$$

$$= \frac{E_m}{N} e^{-i\omega t} e^{ikr_1} \left(\frac{1 - e^{i2\alpha}}{1 - e^{i2\alpha/N}} \right)$$

$$\vec{E}_0 \cdot \vec{E}_0^* = \frac{E_m^2}{N^2} \left(\frac{1 - e^{i2\alpha}}{1 - e^{i2\alpha/N}} \right) \left(\frac{1 - e^{-i2\alpha}}{1 - e^{-i2\alpha/N}} \right)$$

$$= \frac{E_m^2}{N^2} \frac{2 - e^{i2\alpha} - e^{-i2\alpha}}{2 - e^{i2\alpha/N} - e^{-i2\alpha/N}} = \frac{E_m^2}{N^2} \frac{1 - \cos 2\alpha}{1 - \cos \frac{2\alpha}{N}}$$

$$= \frac{E_m^2}{N^2} \frac{\sin^2 \alpha}{\sin^2 \frac{\alpha}{N}} \quad \text{当 } N \text{ large } \sin \frac{\alpha}{N} \sim \frac{\alpha}{N}$$

$$E_m = \phi R = E_m^2 \frac{\sin^2 \alpha}{\alpha^2}$$

$$\alpha = \pi \left(\frac{a}{\lambda} \right) \sin \theta$$

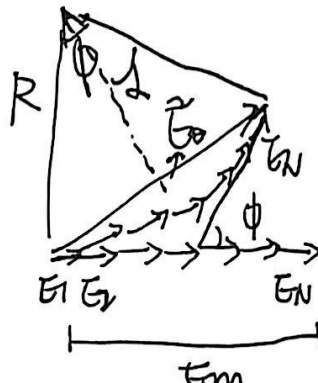
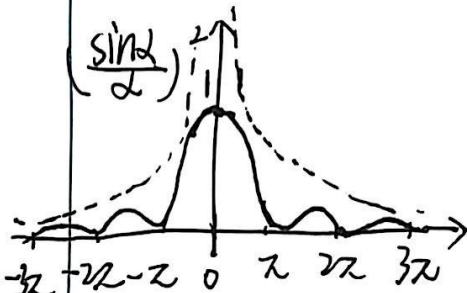
$$\phi = (N-1) \frac{a}{N} k \sin \theta$$

$$NR \text{ 足够大时 } \phi = ka \sin \theta$$

$$\phi = 2\alpha$$

计算和: 余弦定理

$$E_0^2 = R^2 + R^2 - 2R^2 \cos \phi = 2R^2(1 - \cos \phi) = 4R^2 \sin^2 \frac{\phi}{2}$$



2024

$$\Delta x = \frac{a}{N}$$

黎曼和近似 7月

Sunday

农历六月初二
星期日

$$\tilde{E}_0 = \frac{E_0}{N} e^{-i\omega t} \sum e^{ikR_i} = \frac{E_0}{a} e^{-i\omega t} \sum e^{ikR_i}$$

对x求和

a function of k_x

$$N \rightarrow \infty \quad \frac{E_0}{a} e^{-i\omega t} \int_0^a e^{ik(R_1 + x \sin \theta)} dx = \frac{E_0}{a} e^{ik(R_1 - \omega t)}$$

$$\int_{-\infty}^{\infty} e^{ik_x x} dx = e^{ik(R_1 - \omega t + k_x \frac{a}{2})} \frac{E_0}{a} \int_{-\frac{a}{2}}^{\frac{a}{2}} e^{ik_x x} dx \quad k_x = k \sin \theta$$

$$F(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx$$

$$f(x) = \int_{-\infty}^{\infty} F(\omega) e^{i\omega x} d\omega$$

系数与复共轭无影响

one-dimensional aperture function 光阑函数

$$E_{sq}(x) = \begin{cases} E_0 & |x| \leq \frac{a}{2} \\ 0 & |x| > \frac{a}{2} \end{cases}$$

$$\text{对 } E_{sq}(x) \text{ 傅里叶变换 } \tilde{E}_{sq}(k_x) = \int_{-\infty}^{\infty} E_{sq}(x) e^{ik_x x} dx$$

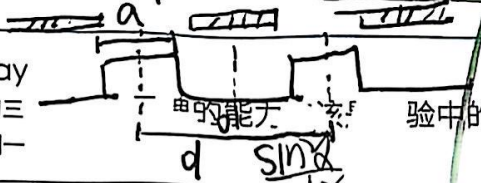
$$= E_0 \int_{-\frac{a}{2}}^{\frac{a}{2}} e^{ik_x x} dx$$

The Fourier Method for the Double slit $E_{ds} = \begin{cases} 0 & \text{otherwise} \\ E_0 & \frac{a}{2} \leq |x| \leq \frac{3a}{2} \end{cases}$

8

Monday

农历六月初三
星期一

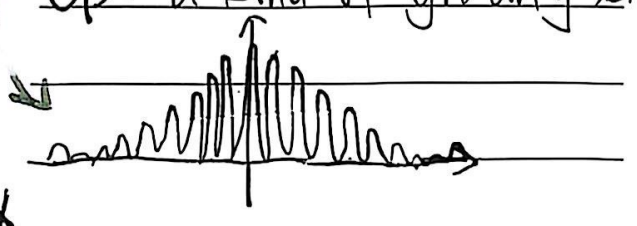
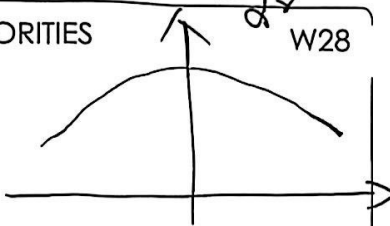


$$\tilde{E}_{ds}(k_x) = \int_{-\infty}^{\infty} E_{ds} e^{ik_x x} dx$$

CP: a kind of grating 光栅

本周计划 TOP PRIORITIES

- ☐ 若 $a = \lambda$.
- ☐
- ☐ a diffraction pattern shrinks.
- ☐ 中央相对高度仍为1.
- ☐ 对于 $\int_0^a e^{ik_x x} dx = \frac{1}{ik_x} [e^{ik_x a} - 1]$
- ☐ $= \frac{e^{ik_x a/2}}{ik_x} [e^{ik_x a/2} - e^{-ik_x a/2}]$
- ☐ $= \frac{e^{ik_x a/2}}{ik_x} 2i \sin \frac{k_x a}{2} = \tilde{E}_0$

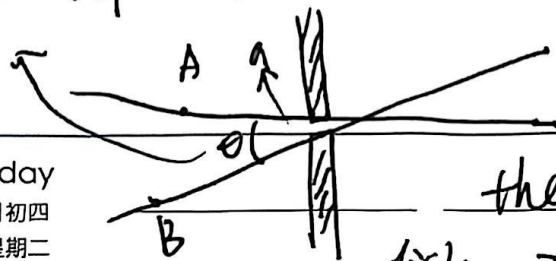


相比较大光圈: 更多光线, 仅在某距离清晰, 其余模糊. 小光圈: 不管距离如何均清晰

Rayleigh's criterion for resolvability θ_R : angular separation - 光源可分辨图像的中心最大值与另一光源可分辨图像

$$I_0 = \tilde{E}_0 \tilde{E}_0^* = \frac{\sin^2(\frac{a}{\lambda} \sin \theta)}{(\frac{a}{\lambda} \sin \theta)^2} = \frac{\sin^2 \theta}{\theta^2}$$

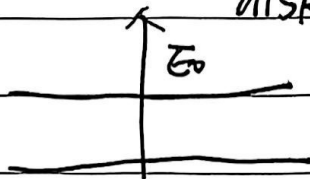
原因: 维 $\theta_R = \sin^{-1} \frac{1.22\lambda}{a} \approx \frac{1.22\lambda}{a}$



$\theta_{1st} = \sin^{-1} \frac{\lambda}{a}$ (瑞利判据)

the Fourier transform of a disk $\rightarrow$ image called Airy Pattern

☐ 当光函数



即 $a \rightarrow \infty$

$$E = E_0 \int_{-\infty}^{\infty} e^{ikx} dx = E_0 2\pi \delta(kx)$$

$\int_{-\infty}^{\infty} f(x) \delta(x) dx = f(0)$

对常数傅里叶变换得到 δ 函数



$$\tilde{E}(k_x) = \int_{-\frac{a}{2}}^{\frac{a}{2}} E_0 e^{ik_x x} dx + \int_{\frac{a}{2}}^{\frac{a}{2}+a} E_0 e^{ik_x x} dx$$

$$= \int_{-\frac{a}{2}}^{\frac{a}{2}} E_0 e^{ik_x (x' - \frac{a}{2})} dx' + \int_{\frac{a}{2}}^{\frac{a}{2}+a} E_0 e^{ik_x (x' + \frac{a}{2})} dx'$$

$$= (e^{-ik_x \frac{a}{2}} + e^{ik_x \frac{a}{2}}) \int_{-\frac{a}{2}}^{\frac{a}{2}} E_0 e^{ik_x x'} dx'$$

双缝干涉

单缝衍射

对双缝的傅里叶变换

Lecture 19 Grating and Spectra $\rightarrow$ 光谱

干涉部分可用卷积理解

☐ $F[f_1(x) * f_2(x)] = F_1(\omega) F_2(\omega)$ 卷积的变换为变换的乘积

☐ output $\int_{-\infty}^{\infty} f_1(\xi) f_2(x-\xi) d\xi$ $E_d(x) = \int_{-\infty}^{\infty} E_{sq}(x') h(x-x') dx'$

☐ 简记 $f_1 \otimes f_2$ input. $F[E_d(x)] = F[E_{sq}(x)] F[h(x)]$

杨氏双缝的光圈函数: E_{sq} 与 h 为两[光圈]函数



$h(x) = \delta(x + \frac{a}{2}) + \delta(x - \frac{a}{2})$

$F(h(x)) = \int_{-\infty}^{\infty} h(x) e^{ik_x x} dx = e^{-ik_x \frac{a}{2}} + e^{ik_x \frac{a}{2}}$

$= 2 \cos(k_x \frac{a}{2}) \quad I = 4 \cos^2(k_x \frac{a}{2})$



: square function 与 $2\pi \delta$ 函数的卷积

对于有宽度 a 的杨氏干涉: $I(\theta) = I_{max} (\frac{\sin \alpha}{\alpha})^2 \cdot \cos^2 \beta$

$\beta = \frac{\pi d \sin \theta}{\lambda}$

最暗处位置

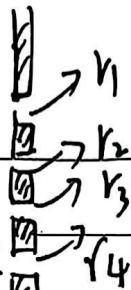
$\alpha = n\pi, \beta = (n + \frac{1}{2})\pi$

2024

7月

11

Thursday
农历六月初六
星期四



$$I(\theta) = \frac{\sin^2 \alpha}{\sin^2 \frac{\alpha}{N}} \quad \alpha = \frac{N d \sin \theta}{\lambda}$$

$$= \frac{\sin^2 \left(\frac{N}{2} k d \sin \theta \right)}{\sin^2 \left(\frac{k d}{2} \sin \theta \right)} \quad \text{记为 } \frac{\sin^2(N\phi)}{\sin^2 \phi}$$

☐

对 $\frac{\sin^2 N\phi}{\sin^2 \phi}$ 作图

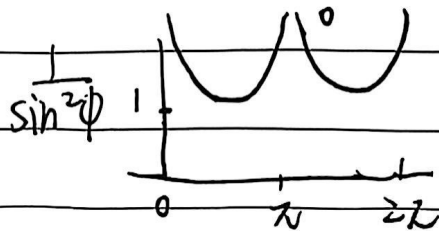
$\sin^2 N\phi$ 零点 $\frac{n\pi}{N}$ $\phi=0$ 时为 N^2

☐

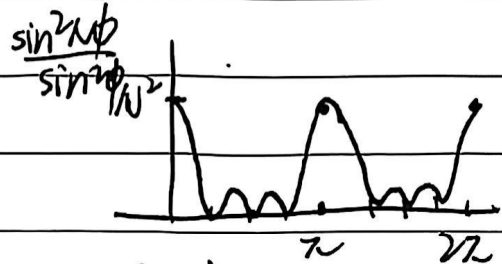
故在 $\pi, 2\pi, \dots$ 处也为 N^2 .

☐

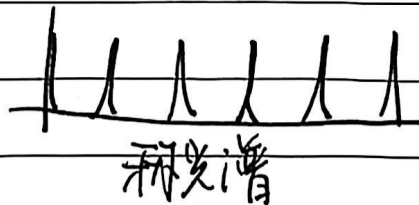
$N=2$ 时



$N=4$ 时



对光栅当 N 很大时.



$\frac{k d \sin \theta}{2} = m\pi$ 时有峰
 $d \sin \theta = m\lambda$
称光谱

波长越长光谱越宽.

宇宙中来自某恒星的星光含原子

12

Friday
农历六月初七
星期五

命运有些随机, 但命运在挑选.

光谱会 shift 由于波长长于地球

光栅由 crystals 构成.

☐

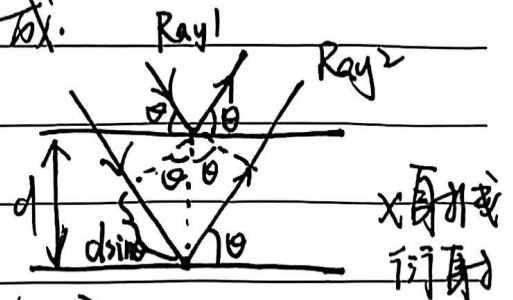
用极化平面的角度来测量

☐

Ray 2 额外距离决定了干涉

☐

$$\Delta x = d \sin \theta$$



Bragg's law

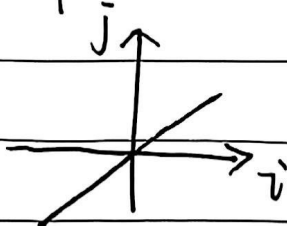
X 射线衍射的强度最大处 $2d \sin \theta = m\lambda$

Lecture 20 Polarization

transverse wave 横波 longitudinal wave 纵波

Example 1 $\vec{E}(z, t) = E_0 x \cos(kz - \omega t) \hat{i} + E_0 y \cos(kz - \omega t) \hat{j}$

linear polarization



在图示线上振荡

2024

★理论上每次转动无限小角度直至 90° 结果为无衰减改变方向

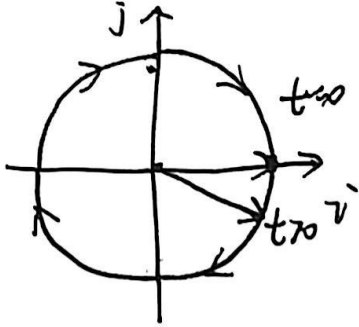
July

13

Saturday
农历六月初八
星期六

Example 2 $\vec{E}(z, t) = E_0 \cos(kz - \omega t) \hat{i} + E_0 \sin(kz - \omega t) \hat{j}$

$= E_0 \cos(kz - \omega t) \hat{i} + E_0 \sin(kz - \omega t) \hat{j}$. $E_0y = E_0x$.



Circular polarization

顺时针 right-circular light.

for $\vec{E} = E_0 \cos(kz - \omega t) \hat{i} - E_0 \sin(kz - \omega t) \hat{j}$

逆时针 left-circularly polarized

A linear polarized wave 可由 2 个 反向偏振的 circular waves of equal amplitude 合成.

$|H\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ horizontal $|V\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ vertical $|D\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ diagonal 斜角 $|AD\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ antidiagonal

14

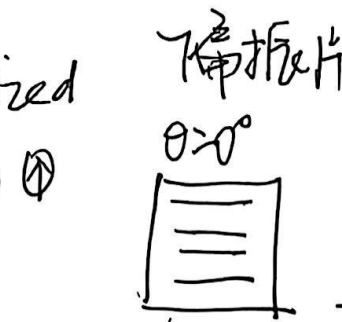
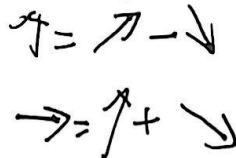
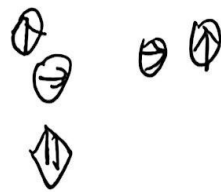
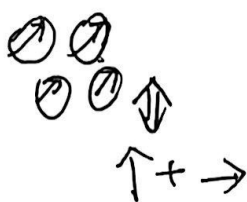
Sunday
农历六月初九
星期日

$|R\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$ Right-circular light

$|L\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$ $\sqrt{\begin{pmatrix} 1 \\ 0 \end{pmatrix} + i \begin{pmatrix} 0 \\ 1 \end{pmatrix}}$

太阳为偏振光.

polarized versus unpolarized



两种均 50% 通过

① 均通过
② 50% 通过

Lecture 21 The Quantum Nature of Light

light come in packets

$h = 2\pi\hbar$

光电效应

$E = hf = \hbar\omega$

$v = f\lambda$

$E = cp$ (from special relativity)

light has momentum . behaves like particle

15

Monday
农历六月初十
星期一

全由自己决定成为什么人，去到哪里，没有一个人能真正左右和笼罩另一个人。

本周计划 TOP PRIORITIES

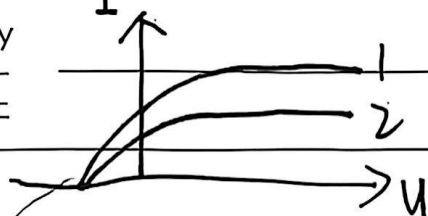
W29

- ☐ $p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$
 - ☐ 德布罗意 $\lambda = \frac{h}{p}$
 - ☐ 对 matter $\vec{p} = \gamma m \vec{v}$ ↑
乘点质量
 - ☐ $\gamma = 1 - \frac{v^2}{c^2}$ $E = \gamma mc^2 = E_0 + \frac{1}{2}mv^2 + \dots$
 - ☐ particles behave like wave
 - ☐ eg. 电子 me. 速度 300 m/s.
 - ☐ $\lambda \sim 10^{-6} \text{ m}$
 - ☐ 对于光 $m=0$ (photon. $v=c$)
- The photoelectric effect:

光的频率超过极限频率
时有光电流
入射光一定时增大光电管两
极的正向电压，提高光电子
动能，光电流增大至饱和电流
入射光强度越大光电流也越大
遏止电压与入射光 f 有关，与
光强无关。 线性
当入射光的 f 减小到某一数
值，即使不加反向电压也无光
电流，称有截止频率或有极限频率。

16

Tuesday
农历六月十一
星期二



光强: $1 > 2$.

$$hf = K + W_0 \rightarrow \text{逸出功.}$$

$$V_{\text{stop}} = \frac{h}{e} f - \frac{1}{e} W_0$$

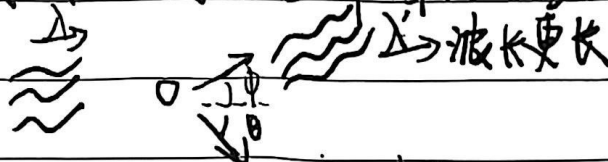
☐

☐

$$E^2 = c^2 p^2 = m^2 c^4 \quad \text{for photon } m=0 \quad E=cp$$

☐

康普顿散射: 一种光子与物质相互作用的过程，其中光子与物质中的电子发生非弹性碰撞，导致光子能量降低并散射。



$$m_1 \rightarrow m_2 \quad 90^\circ \text{ when } m_1 = m_2$$

能量守恒 $\frac{hc}{\lambda} + mc^2 = \frac{hc}{\lambda'} + \gamma mc^2 \rightarrow \frac{h}{\lambda} - \frac{h}{\lambda'} = mc + \gamma mc$

动量守恒 $\frac{h}{\lambda} = \frac{h}{\lambda'} \cos \phi + \gamma m v \cos \theta$

$$0 = \frac{h}{\lambda'} \sin \phi + (\gamma m v \sin \theta)$$

$$\Delta \lambda = \frac{h}{mc} (1 - \cos \phi)$$

若自旋则有角动量

光子有角动量。

右旋或左旋
right left 旋转

固有的

17

Wednesday
农历六月十二
星期三

每当电子发射或吸收电磁辐射时, 电子
会发生能级跃迁。

☐ Lecture 22 Matter Waves

☐ probability $\propto$ (probability amplitude)²
☐

$$|\psi|^2$$

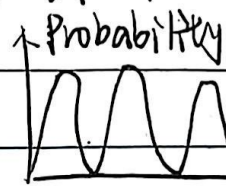
↓
wave function

$$\text{Probability } P = |\psi|^2 = |\psi_1|^2 + |\psi_2|^2 + 2\text{Re}(\psi_1^* \psi_2) + \dots$$

干涉项

Heisenberg's Uncertainty Principle

位置与动量不能同时知道: $\Delta x \cdot \Delta p_x \geq \hbar/2$ 其他方向同理
电子干涉图样



$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$$

ψ : probability amplitude

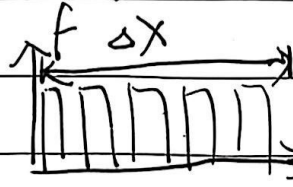
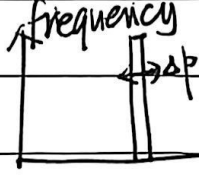
18

Thursday
农历六月十三
星期四

$$\text{Variance 方差 } \Delta A = \overline{(A - \bar{A})^2} = \overline{A^2} - \bar{A}^2$$

☐

若 p fixed



can be found

everywhere

i.e. perfect wave

$$\psi_p(x) = e^{i(kx - \omega t)}$$

$$p = \frac{h}{\lambda} = \frac{h}{2\pi} k = \hbar k$$

$$\Psi(x) = \sum_p C_p \psi_p(x) = \sum_k C_k e^{i(kx - \omega t)} \quad \text{i.e. 傅里叶变换}$$

$$\psi(x) = \delta(x - x_0) \quad \bar{\psi}(p) = \bar{f}(\psi(x)) = \text{constant}$$

↑
probability = $|\psi(x)|^2$

Schrodinger's equation governs the $\psi(x)$.

Lecture 23 Schrodinger's equation 薛定谔方程

A classical wave $\psi(x, y, z, t)$ satisfies the wave equation

$$\frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} = \nabla^2 \psi$$

19

Friday
农历六月十四
星期五

时间和人都不会在一个地方停留太久，总有变化推着人再次出发，去赴未来的命运。

$$\psi(x,t) = e^{i(kx - \omega t)}$$

$$E = \frac{p^2}{2m} \rightarrow \frac{mv^2}{2} \text{ 动能}$$

□

$$\frac{d\psi}{dx} = ik e^{i(kx - \omega t)} \quad \hbar \frac{d\psi}{dx} = i\hbar k e^{i(kx - \omega t)}$$

□

$$\hat{p} = \frac{\hbar}{i} \frac{\partial \psi}{\partial x} \quad \hat{p}^2 = -\hbar^2 \frac{\partial^2 \psi}{\partial x^2} \cdot \psi$$

□

$$\frac{\partial \psi}{\partial t} = -i\omega e^{i(kx - \omega t)} \quad E = \hbar\omega \quad i\hbar \frac{\partial \psi}{\partial t} = E\psi$$

$$i\hbar \frac{1}{\psi} \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{1}{\psi} \frac{\partial^2 \psi}{\partial x^2} \quad \text{最简单的薛定谔方程}$$

更一般的情况:

for free particle 自由粒子, in free space

$$E = \frac{p^2}{2m} + U \rightarrow \text{势能} \quad i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + U\psi$$

物质波遵循薛定谔方程

$$\psi(x,t) = \phi(x) e^{-iEt/\hbar}$$

$$i\hbar \frac{\partial \psi}{\partial t} = E\phi(x) e^{-iEt/\hbar}$$

$$\left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + U\right)\psi = E\phi(x) e^{-iEt/\hbar} \quad \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + U\right)\phi = E\phi(x)$$

20

Saturday
农历六月十五
星期六
 $M\vec{x} = \lambda\vec{x} \rightarrow \text{特征值}$
 $\downarrow \quad \downarrow$
 矩阵 特征值 向量
爱因斯坦 $E = \hbar\omega$ for $U(\vec{r}) \propto |\vec{r}|$. 库仑势 此时薛定谔方程描述氢原子。 $\propto x^2$ 谐振势。

阶跃势 step potential

discrete 离散谱, 不连续谱。

the general solution for 薛定谔方程 $\Psi(x,t) = \sum_{n=1}^N c_n \phi_n(x) e^{-i\epsilon_n t/\hbar}$ for free particle $U=0$ $-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi(x) = E \Psi(x)$ $\phi_{\pm}(x) = A_{\pm} e^{\pm i\sqrt{\frac{2mE}{\hbar^2}} x}$

$$\Psi(x,t) = A e^{i\sqrt{\frac{2mE}{\hbar^2}} x - i\epsilon t/\hbar} + B e^{-i\sqrt{\frac{2mE}{\hbar^2}} x - i\epsilon t/\hbar} = A e^{i(kx - \omega t)} + B e^{-i(kx - \omega t)}$$

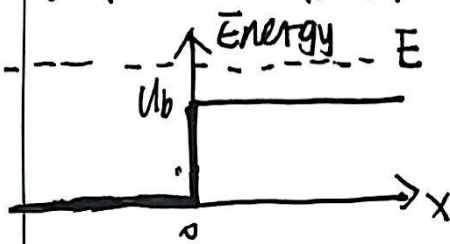
$$|\Psi(x,t)|^2 = \Psi^*(x,t) \Psi(x,t)$$

21

Sunday
农历六月十六
星期日

一束总能量为 E 的相对论性电子, 沿着 x 轴

穿过一狭管, 在 $x=0$ 处分布 $V_b < 0$ 的反电势阱, 考虑 $E > qV_b$ 电子应全部通过边界. 将薛定谔方程分别应用于这两区域. $x=0$ 处波函数在值与斜率上均应一致 (连续 + 光滑 + 归一化)



$$x < 0 \text{ 时 } k = \sqrt{2mE}/\hbar \quad \psi_1 = Ae^{ikx} + Be^{-ikx}$$

$$x > 0 \text{ 时 } k_b = \sqrt{2m(E - qV_b)}/\hbar$$

$$\psi_2 = Ce^{ik_b x} + De^{-ik_b x}$$

设 $D=0$ (由于右边无电子源)

$x=0$ 的边界条件:

$$A+B=C$$

$$Ak - Bk = Ck_b \quad (\text{斜率匹配})$$

为求阱深及反射概率

$$\text{反射系数 } R = \frac{B^2}{A^2} = \frac{(k - k_b)^2}{(k + k_b)^2}$$

$$\text{透射系数 } T = 1 - R = \frac{4kk_b}{(k + k_b)^2}$$

22

Monday
农历六月十七
星期一

人本身最美, 运动最时尚, 健康最奢侈。

$$\frac{C^2}{A^2} = \frac{4k^2}{(k + k_b)^2} = T \frac{k}{k_b}$$

$$\text{电流密度 } J = nqV$$

$$p = \hbar k = mv$$

$$J_{\text{transmitted}} = \frac{J_{\text{transmitted}}}{J_{\text{incident}}}$$

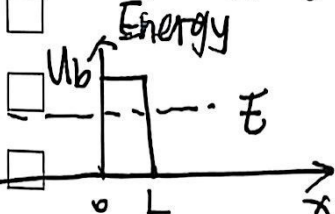
$$V = \frac{\hbar k}{m}$$

本周计划 TOP PRIORITIES

$$T = \frac{C^2 k_b}{A^2 k} = \frac{0^2 q \hbar k_b / m}{A^2 q \hbar k / m}$$

$$R = \frac{J_{\text{reflected}}}{J_{\text{incident}}}$$

When $E < qV_b$



隧穿效应:

电子等微观粒子能穿过

入或穿越位势垒的量子行为, 尽管位势垒的高度大于粒子的总能量. 在 $0 < x < L$ 处

为指数衰减函数

$$x < 0 \quad \psi_I = Ae^{ikx} + Be^{-ikx}$$

$$0 < x < L \quad \psi_{II} = Ce^{Kx} + De^{-Kx}$$

$$x > L \quad \psi_{III} = Fe^{ikx}$$

$$k = \sqrt{\frac{2mE}{\hbar^2}} \quad K = \sqrt{\frac{2m(qV_b - E)}{\hbar^2}}$$

$$T \approx e^{-2KL}$$

$$\text{研究 } \frac{B^2}{A^2} \cdot \frac{|E|}{T}$$

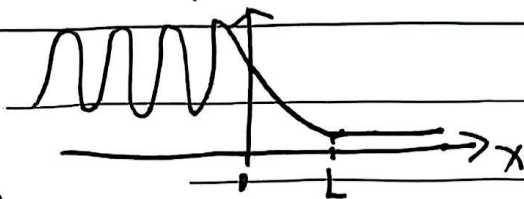
$x < 0$ 处为入射波 (频率相同, 传播方向相反) 和反射波

2024

probability density $|\psi(x)|^2$

7月

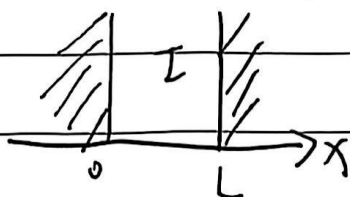
23

Tuesday
农历六月十八
星期二 $\frac{E}{A} \rightarrow \text{tiny}$

□ Lecture 24 Quantum Wells.

$$\square \nabla^2 \psi = -\frac{2m}{\hbar^2} (E - U(x, y, z)) \psi(x, y, z) = E \psi(x, y, z)$$

□ 一维无限势阱

 $V \rightarrow -\infty \quad V=0 \quad V \rightarrow -\infty$ $x=0 \quad x=L$ 电子可以被困在
 $V=0$ 区.

region I : $-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \psi(x) = E \psi(x)$

$$\psi(x) = A e^{i \sqrt{\frac{2mE}{\hbar^2}} x} + B e^{-i \sqrt{\frac{2mE}{\hbar^2}} x} \quad \dots (*)$$

$$\psi(0) = \psi(L) = 0 \quad = A' \sin \sqrt{\frac{2mE}{\hbar^2}} x + B' \cos \sqrt{\frac{2mE}{\hbar^2}} x$$

$$B' = 0$$

$$\psi(x) = A' \sin \sqrt{\frac{2mE}{\hbar^2}} x = A' \sin Kx$$

$$\psi(L) = A' \sin KL$$

$$KL = n\pi \quad K = n \frac{\pi}{L}$$

□

□

for (*) $A+B=0 \quad B=-A$

$$\psi(x) = A \left(e^{i \sqrt{\frac{2mE}{\hbar^2}} x} - e^{-i \sqrt{\frac{2mE}{\hbar^2}} x} \right)$$

$$\therefore \psi_n(x) = A_n \sin(k_n x) \quad k_n = \frac{n\pi}{L} = \sqrt{\frac{2mE}{\hbar^2}}$$

$$E_1 = \frac{\pi^2 \hbar^2}{2mL^2} n^2$$

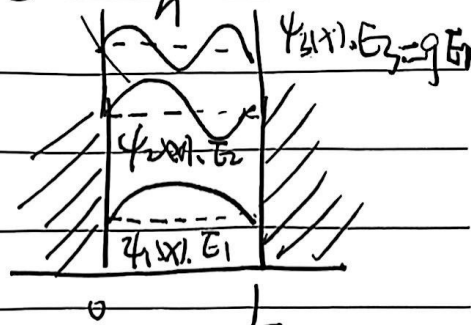
$$\psi_1 = A_1 \sin\left(\frac{\pi}{L} x\right)$$

$$E_1 = \frac{\pi^2 \hbar^2}{2mL^2}$$

$$E_2 = 4 E_1$$

$$E_3 = 9 E_1$$

⋮

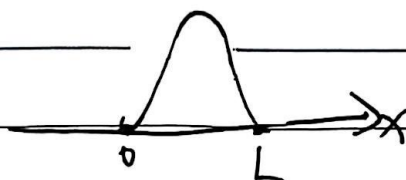
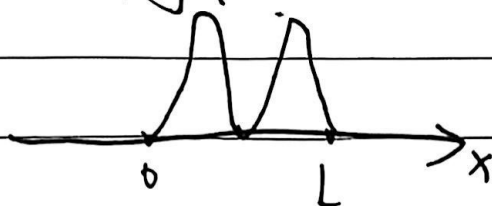


$$\lambda = 2L$$

以驻波形式存在.

 E_1 为 groundstate E_2 为 1st excited state

25

Thursday
农历六月二十
星期四probability $|\psi(x)|^2$ ☐ $|\psi(x)|^2$ ☐☐

$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$. 由此 $\psi(x) = A \sin kx$ 可确定 k .

归一化条件

$$\therefore A = \sqrt{\frac{2}{L}}$$

Dimension of $\psi(x)$ $[\psi(x)] = \frac{1}{\sqrt{[L]}}$

二维 $[\psi(x,y)] = \frac{1}{\sqrt{\text{Area}}}$

三维 $[\psi(x,y,z)] = \frac{1}{\sqrt{\text{Volume}}}$

在中心圆阱中运动的电子总能量为 $E_n = \left(\frac{h^2}{8mL^2}\right)n^2$

正整数 n 是 quantum number of the electron's

quantum state in the trap. 二维 $E_n = \frac{\pi^2 \hbar^2}{2mL^2} n^2$

26

Friday

农历六月廿一

那些在自我的修罗场中争斗过，不死的才叫做信念。

$$\hbar = \frac{h}{2\pi}$$

Lecture 15 The Hydrogen Atom $E_n = \frac{h^2}{8mL^2} n^2$

☐

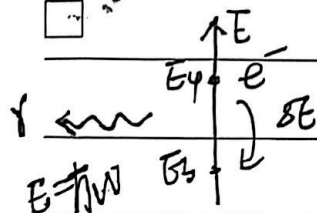
$n=0$. $n=0$ 时 $\psi=0$. 阱中无电子. 全空.

☐

受限系统必须始终具有一定的最小能量, 称为零点能量

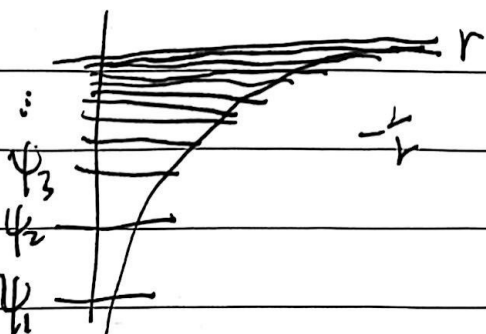
☐

氢原子薛定谔方程:



$$\left[-\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) - \frac{e^2}{r} \right] \psi(x,y,z) = E \psi(x,y,z)$$

Coulomb potential



$$E_n \propto -\frac{1}{n^2}$$

$$E_1 = -13.6 \text{ eV}$$

Lowest $\neq -\infty$

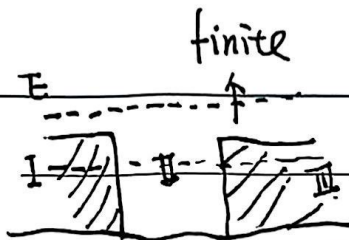
故电子不会塌缩到核子上

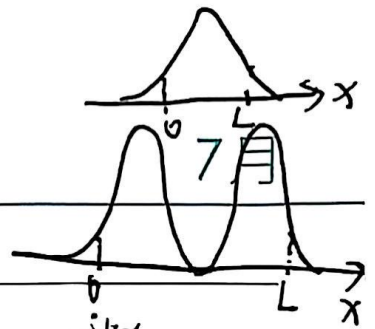
Correspondence Principle: n 足够大时, 量子物理学的预测与经典物理学的预测光滑融合.

2024

27

Saturday
农历六月廿二
星期六

finite for $E < U_0 \Rightarrow$

 $\psi_I(x) = Ae^{ikx}$
 $\psi_{II}(x) = Be^{ikx} + Ce^{-ikx}$
 $\psi_{III}(x) = De^{-\kappa x}$



2D: $\psi(x,y) = -\frac{\hbar^2}{2m} \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right] \psi(x,y)$

令 $\psi(x,y) = X(x)Y(y)$

$$E = -\frac{\hbar^2}{2m} \frac{1}{X(x)} \frac{\partial^2 X(x)}{\partial x^2} - \frac{\hbar^2}{2m} \frac{1}{Y(y)} \frac{\partial^2 Y(y)}{\partial y^2}$$

$$\psi_n(x,y) = \frac{2}{\sqrt{L_x L_y}} \sin\left(\frac{n_x \pi}{L_x} x\right) \sin\left(\frac{n_y \pi}{L_y} y\right)$$

$$E_{n_x, n_y} = \frac{\hbar^2}{8m} \left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} \right)$$

泡利不相容原理: 一个轨道最多容纳2个电子且自旋相反

no two electrons confined to the same trap can have the same set of values for their quantum numbers.

28

Sunday
农历六月廿三
星期日

(若自旋相反则可以)

e.g. $e_1 = \psi_{n_x=1, n_y=1} \uparrow$ $e_2 = \psi_{n_x=1, n_y=1} \downarrow$ ✓

when $L_x = L_y = L$.

能量量子化.

$$E \sim n_x^2 + n_y^2$$

$$n_x=1, n_y=1$$

$$E \sim 1^2 + 1^2 = 2$$

同一能量不同状态.

① $n_x=2, n_y=1$

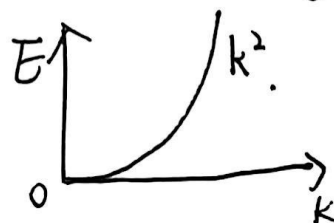
$$E \sim 5$$

② $n_x=1, n_y=2$

$$E = \frac{p^2}{2m} + U \text{ for free particle}$$

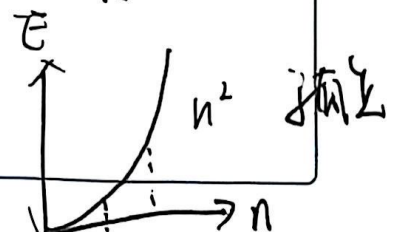
$$E = \frac{p^2}{2m}$$

$$p = \hbar k = \hbar \frac{2\pi}{\lambda}$$



能量可取 $(0, \infty)$

infinite potential well: $E_n = \frac{\hbar^2}{8mL^2} n^2$



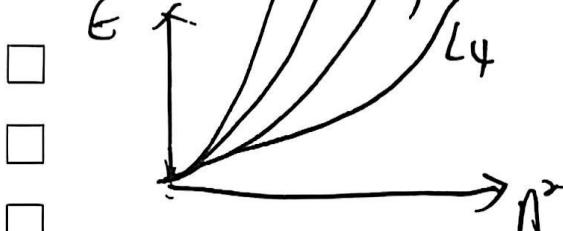
29

Monday
农历六月廿四
星期一

好身材的内里是运动员精神, 是经历惊慌、困惑、悲伤和恐惧之后的勇气。

when L increases. $E < U_b$ 时

本周计划 TOP PRIORITIES



W31

$$L_1 < L_2 < L_3 < L_4$$



L 越来越大, ΔE 越来越小.

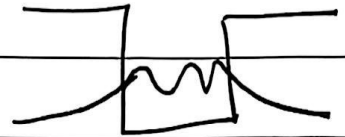
when $L \rightarrow +\infty$.

接近于连续能量情形

Quantum well.



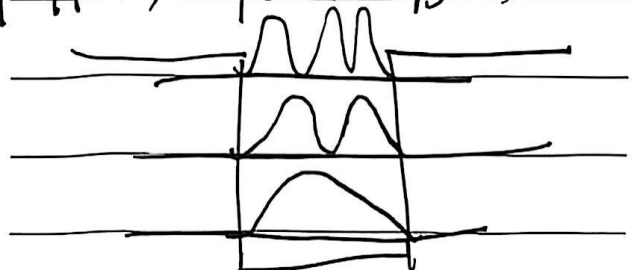
连续.



为满足边界条件, n 有限.

quantum number n 越大, leakage 越大.

$$\text{即 } \psi_1^2(0) < \psi_2^2(0) < \psi_3^2(0) \dots$$

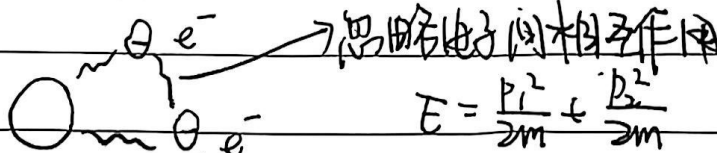


likely to be found everywhere

30

Tuesday
农历六月廿五
星期二

对于双原子如 He.



$$\Psi_{n,n'}(x_1, x_2) = \psi_n(x_1) \phi_{n'}(x_2)$$

Wave function